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IN JAPAN

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ELEMENTARY PRINCIPLES
OF
THE STATISTICAL CONTROL
OF
QUALITY

—A Series of Lectures—

By
W. Edwards Deming. Ph. D.

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PREFACE

To

The Revised Second Edition

This is the revised second edition of "Dr. W. Edwards Deming's Lectures on Statistical Control of Quality, 1950, Tokyo." Based on the shorthand notes taken by Japanese stenographers and put to press without being revised by the author due to unavoidable circumstances, the first edition left much to be desired. But it enjoyed great popularity among Japanese quality control engineers and was soon out of print. After June, 1951, inquiries about the second printing came increasingly to the Union of Japanese Scientists and Engineers, but the author, referring to the errors and misinterpretations in the first edition, requested that no more copies be published without his personal revision. Fortunately, Dr. Deming again visited Japan in the summer of 1951, and in January, 1952, for series of lectures on quality control and market research. In spite of the pressure of work during his short stay, he carefully looked over the manuscripts, making necessary corrections, deletions and additions from page to page, so much so that an entirely new edition like this had to be published.

Mr. Hisamichi Kano corrected the proofs, and collaborated with the editor to match the Japanese translation properly with its English original.

In accordance with the author's advice, the title has been changed to "Elementary Principles of the Statistical Control of Quality……A Series of Lectures by Dr. W. Edwards Deming."

10 April 1952

Ken-ichi Koyanagi

EDITOR'S PREFACE

This is a shorthand transcript of Dr. W. Edwards Deming's lectures on "Statistical Control of Quality."

As everybody knows, Dr. Deming is one of the world-famous authorities on the theory of sampling. In the field of statistical quality control, i. e. application to industrial production of mathematical statistics, too, he is versed in both theory and practice. This is clear from the fact that he has published many papers and books, especially notable his *Some Theory of Sampling*. It was he who edited W. A. Shewhart's *Statistical Method from Viewpoint of Quality Control* (1939). As a member of the five-man Emergency Technical Committee, he played a major role, in collaboration with Messrs. H. F. Dodge, A.G. Ashcroft, Leslie E. Simon, R.E. Wareham and John Gaillard, in the compilation of the American War Standards (ASA ZI.1-3). He has since been striving in earnest for wider application of this method; and Mr. W. B. Rice and other experts in quality control dedicate their books to Dr. Deming (for instance, Rice's *Control Charts in Factory Management*, 1947).

Therefore, Dr. Deming had been the idol of Japanese students in quality control. Fortunately, when he revisited Japan in the summer of 1950, he kindly accepted our request for a lecture course and gave a series of lectures on statistical control of quality for eight days (10-18 July) at the Nippon Ishikaikan Hall, Kanda, Ochanomizu, Tokyo, to 230 first-class engineers of leading manufacturing corporations. His lectures were of the same standard as those he gave in the United States where statistical quality control methods have made remarkable progress since wartime. Throughout the 8-day course there always prevailed an atmosphere of unity between the teacher and students. With simple explanations and adequate demonstrations, Dr. Deming's lectures were so effective and persuasive that they left an unforgettable

impression upon our minds.

With a view to attaining satisfactory results in the projected lecture course, the Union of Japanese Scientists and Engineers had made careful preparations:

1) Mr. Hisamichi Kano was selected as interpreter. As Dr. Deming himself recognized with thanks, Mr. Kano is an interpreter of rare talent and knowledge in English.

2) Many Japanese scientists and GHQ officials were so kind as to make comments on Dr. Deming's lectures or deliver supplementary lectures. They were, among others: Dr. Motosaburo Masuyama, Dr. Eizaburo Nishibori, Dr. Shigeru Mizuno, Mr. Heihachi Sakamoto, Mr. H. M. Sarasohn, Mr. W. MacPhee, Mr. Kaoru Ishikawa, Mr. Masao Kogure, Mr. Shin Miura, Mr. Eizo Watanabe, and Mr. Masao Goto.

3) Mr. Susumu Sonobe, of the Nippon Denki K. K., volunteered to take care of the public address system in the Lecture Hall.

4) In order to make a perfect record of Dr. Deming's lectures, a recording subcommittee was formed by Dr. Mizuno (as chairman), Mr. Ikuro Kusaba, Mr. Tadasu Fujita and Mr. Masumasa Imaizumi. Both Japanese and English stenographers were employed. We thought it essential to make a perfect record of Dr. Deming's authoritative lectures because there had been in Japan some sort of confusion and disorder in the application to industry of statistical quality control methods, which we hoped might be brought to a settlement by Dr. Deming's enlightenment; and inasmuch as such a record would be of great use not only to the students who attended the course but also to many others in manufacturing plants for repeated study of Dr Deming's lectures and the development of new ideas and devices. Fortunately, Dr. Deming allowed us to publish it in book form.

After the lecture course was over, Mr. Kano carefully edited the English shorthand record. In the case of the Japanese edi-

tion, the members of the recording subcommittee lavished efforts for theoretical explanations, mathematical formulas, unification of technical terms, etc. Efforts were made also to effect a unity between the Japanese and English editions.

Not until about three months after Dr. Deming's departure for the United States was our editing work finished. We deeply regret that the English shorthand record was not supervised by Dr. Deming himself, but we think that, thanks to the unstinted efforts on the part of our experts, the record of Dr. Deming's authoritative lectures has been brought to the point of perfection. We are looking forward that Dr. Deming's lectures (the flowing, elegant English edition and the correctly-translated, carefully-worded Japanese edition) will find a large circle of readers among our managers and engineers, and exercise far-reaching influence on the application in Japan of statistical quality control methods, as Lafcadio Hearn's (1850-1904) *Lectures on Literature* were read widely by Japanese writers and authors to give inspiration to the literary world here.

We should like to explain in brief the circumstances in which Dr. Deming kindly consented to give his lectures above explained. In December, 1949, Mr. Sigeiti Moriguti, of Tokyo University, received a letter from Dr. Deming, informing that he would come to Japan in the spring of 1950. On 8 March 1950 we wrote a letter to Dr. Deming, asking that a message be sent to the first issue of our organ "Hinshitsu Kanri" (Statistical Quality Control) for which preparations had almost been completed, and inquiring after the exact date of his arrival here and itinerary. Incidentally, we had been continuing our research works on statistical quality control since 1948 and were holding the SQC Seminar, 30 days a year, attended by 39 engineers of 17 corporations. And our research had developed to the point where we could publish a monthly organ, "Statistical Quality Control."

On 22 March we received a kind letter from Dr. Deming, saying that he would visit this country in the summer of 1950, and asking what kind of article we would like to have. He also recommended the following American scholars to whom our monthly organ should be sent with compliments:

Charles A. Bicking	Dr. Martin A. Brumbaugh
Harold A. Dodge	Dr. E. J. Gumbel
Howard L. Jones	Paul Peach
David H. Schwartz	Dr. Water A. Shewhart
Ralph E. Wareham	Charles D. Wiman
Prof. Mason E. Westcott	Major-Gen. L. E. Simon
Prof. Lloyed A. Knowler	

Deeply moved by the kind letter, on 1 April we wrote again to Dr. Deming, informing him of the title and content of the article which we would like to have. In the same letter, we also asked if he could give a series of lectures on statistical quality control for several days while he was in Tokyo, for this plan had been readily approved by our president, Ichiro Ishikawa, and our lecturers who had been cooperating with us in the SQC Seminar.

We soon received a letter dated 11 April from his secretary saying that Dr. Deming was travelling at that time. Then, a reply of 18 April addressed to Managing Director K. Koyanagi was forwarded to our office from Miss Margaret Stone, PSD, ESS, SCAP. Several days after, an air mail letter dated 22 April almost to the same effect reached us. Enclosed therewith we found "an outline of course in sampling". In these two letters, Dr. Deming kindly accepted our proposal for a lecture course, saying as follows:

"I am deeply honored by your invitation to give some lectures this summer and to assist with your proposed course in the statistical theory of control of quality and its applications. First of all let me say that your plan for an intensive course lasting 5 to

7 days, 7 hours per day, is excellent. …… It will be possible, I am sure, for me to assist you with this course by giving some lectures on statistical theory and demonstrations of applications, from 2 to 4 hours daily. …… In this country we found that 8 days was a minimum for people who had not studied elementary statistical theory. I should think that in Tokyo the 7-day period would probably be much better than the 5-day period. If possible, you might even extend the time to 8 days. ……"

Thereafter, Dr. Deming accorded us all sorts of facility to make effective the projected course. For instance, he recommended as a textbook the *Control Chart Method of Controlling Quality During Production* (ASA Z1.3) and sent us six copies of it; he brought with him equipment necessary for demonstrations of applications; though he was extremely busy in Tokyo, he wrote us a draft for an advanced course; and, above all, he kindly recommended us to Mr. Kenneth Morrow, Chief of PSD, GHQ, SCAP, who helped us in every way to conduct the lecture course with success. Thus, the success of the course was for the most part to the credit of Dr. Deming.

Under these circumstances and thanks to the good offices and cooperation not only of Dr. Deming but also many friends of ours, this transcript of lectures on statistical control of quality has come to be published after having obtained the lecturer's approval. We take this occasion to express our heartfelt thanks to all of them, particularly Mr. Kenneth Morrow, the Japanese scholars who willingly assisted us, and Mr. Shoichiro Niki and other members of our secretariat.

10 November 1950

Ken-ichi Koyanagi
Managing Director, Union of
Japanese Scientists & Engineers

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INTRODUCTION

We are in a new industrial age, created largely by statistical principles and techniques. I shall try to explain how these principles and techniques are helping Japan to increase her export trade.

International trade is an essential component of peace and prosperity. International trade depends on———

a. Quality at the right level (I shall say more on this later)
b. Uniform quality and dependable performance (It is a fact that trade depends more on uniformity than on high levels of quality)
c. Competitive price (which requires efficient production)
d. International standardization
e. Meaningful specifications——tests that will distinguish economically and reliably between good and bad quality
f. Manufacture in sufficient quantity to warrant business
g. International language of communication of quality. (The buyer, even though separated by oceans from the seller, must be able to express the quality desired in a language that both buyer and seller understand. Equally important, the seller must be able to express the quality of the product that he is able to produce and sell, in a language that they both understand. Of course, this problem is not simple, because the needs of the buyer must be met by the product and quality that the producer is capable of making, and for which the buyer has purchasing power.)

Statistical techniques have brought new meaning into all these requirements of international trade. As I said, we are in a new industrial age. Most of you have heard of the quality-control charts and acceptance sampling. The quality-control

chart is a very simple device, much used now in Japan. It is so simple that a shop foreman can learn to use it in a few hours or days, he can and will immediately start to improve the quality and uniformity of his product, and to turn out a greater amount of product every day. His machines will be idle less time every day, and he will waste less material. He will turn out more good product every day, and not so much rejected product. All this he will do without new machines or more space.

The international language that is so essential for the communication between buyer and seller of the quality-characteristics of a product is provided by the same statistical techniques that are used for improved testing, production, and inspection. Those of you who are familiar with the control charts and acceptance sampling will see at once how true this is: the control chart, when the production process has been brought to a state of control, shows indisputable proof of quality and uniformity, to anyone in any part of the world.

Actually, as most of you know, in recent Japanese experience, the simple statistical techniques of the control chart have brought forth increases in production that range from 5% to 100%, *without expansion of plant.* Some firms are saving 10% of their raw material, compared with their performance of a year ago. In some cases, the savings are much greater. One large steel company reported they were able to cut their fuel bill by a third. All these achievements, and more, were reported at the convention held in Osaka the 22nd September, last, at the occasion of the Deming award.

In one large spinning mill that I visited last summer in Japan, three quarters of the girls who had been engaged in rework and repair a year before had been moved into production, because of improved quality and lesser number of rejections.

The gain in production and profit, although considerable from such achievements, are the least of the gains. The gains from a better competitive position through improved quality to lower the price are much greater, though more difficult to measure.

Acceptance sampling is another simple technique, by which quality is guaranteed to the consumer at minimum expense for testing.

In these ways, and in many other ways, statistical techniques help greatly to meet the requirements for increased international trade. I shall now define what I mean by statistical quality control.

Statistical quality control is the application of statistical principles and techniques in all stages of production directed toward the most economic manufacture of a product that is maximally useful and has a market

Now let me tell you what I mean by statistical techniques applied in all stages of production. In any manufacturing plant, raw materials come in, and product flows out. Eventually, your product is used by people who will judge your product. Your raw materials may indeed be raw, or they may be sub-assemblies or piece-parts from another manufacturer. Your finished product may indeed be finished, or it may be raw material for some other manufacturer.

Anyway, there will be a chain of production, and to make

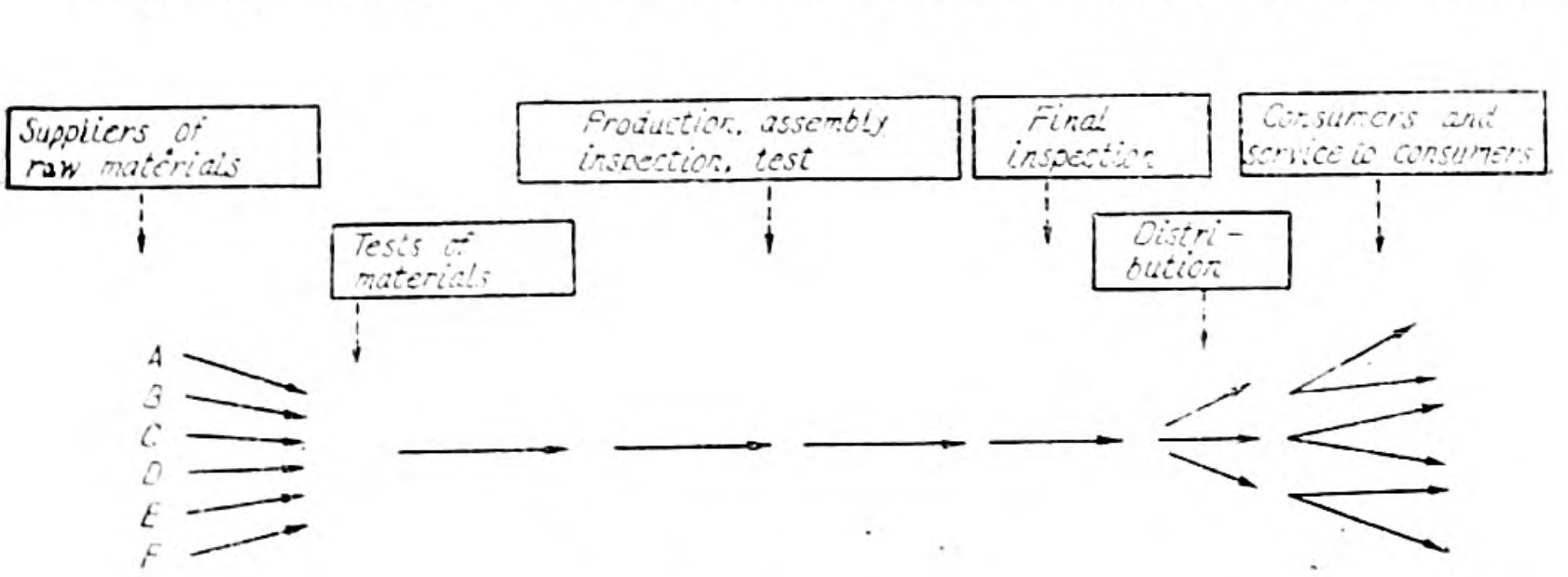


Fig. 1

the problem simple enough for me to deal with in one lecture, I shall draw a diagram (Fig. 1).

By statistical quality control, I mean statistical work in all these stages of production. Statistical techniques are essential in all these stages. It is really impossible for manufacturers to compete and to build up their export trade to the fullest usefulness to themselves and to the rest of the world without using the best techniques: otherwise their quality will not be good enough or uniform enough, or their price will be too high.

Through full use of statistical quality control, from raw material to the consumer, a manufacturer may expect to achieve in some measure the following advantages:

1. Increased production, without investment in capital equipment or expansion of plant
2. Savings on raw materials and fuel (a particularly vital advantage when scarcity of materials threatens production)
3. Better operating efficiency: (a) idle time of machines decreased; fewer rejections; less scrap and re-work; (b) better prediction of the market, through consumer research, by which the purchase of materials, and the expansion and contraction of the plant, are carried out rationally, resulting in better economy than would be possible otherwise
4. Decreased inspection, but with increased assurance of dependable quality
5. Quality, uniformity and price, better suited to the market
6. Greater precision of dimensions when required (as when parts are to be interchangeable)
7. Better design, through consumer research, carried out by modern methods of sampling and design of experiment

4. Stronger competitive position, through ability to meet world requirements in price, quality and uniformity; and to furnish statistical proof of quality and uniformity in an international language.

I must now ask you to think with me in broad terms just what good quality and economic production really mean. Some people think in terms of price alone. Perhaps others think in terms of quality alone. Some people think of economic production as saving 10% in the cost of some operation. All these ideas are important, but we must go deeper.

In the first place, *price has no meaning except in terms of the quality of the product.* But that is not enough. "GOOD QUALITY" *and* "UNIFORM QUALITY" *have no meaning except with reference to the consumer's needs.*

Therefore, next I shall speak to you in terms of the entire production line, which begins with the producers of your raw materials, and ends with the consumer—the man who uses your product.

Incidentally, the consumer is more important than raw material. It is usually easier to replace a supplier of raw material with another one than it is to find a new consumer. And a non-consumer, one who has not yet tried your product, is still more important to you, because he represents *a possible additional user of your product.* For this reason I shall speak to you of *consumer research,* because I believe in the importance of consumer research to Japanese industry, particularly for its export trade.

A moment's digression on the newness of statistical techniques, of statistical quality control, may be interesting. The use of control charts began on a big scale in America only in 1942 and 1943. Acceptance sampling, however, began much

earlier ; about 1926. The techniques for the proper design of tests of raw materials, designs and finished products began about 1920, although their use in industry was not recognized extensively until about 1935. Suitable techniques of sampling and questioning for economical and reliable consumer research have been built only within the last 10 years.

Some manufacturers think of consumer research (or market research) as analysis of complaints from purchasers and users. Certainly, no one can deny the great importance of the analysis of consumers, complain's. No matter how silly and unjust a complaint may be, it is still important to a manufacturer, because it shows him *where he has failed in public relations* — that is, failed to make clear to the public just what quality they have a right to expect when they buy his product. A legitimate complaint helps the manufacturer to improve his quality, provided he has real quality control, and can be used by the manufacturer in tracing the cause of trouble all the way back to production and raw material.

Surely you will agree that complaints are only a part of the problem of public relations—communication with the consumer and the non-consumer.

Complaints come from a very biased sample of consumers. Complaints do not provide communication with the other consumers nor with the non-consumer.

For reliable and economical communication with the consumers and non-consumers of a product, it is necessary to carry out *statistical tests and surveys.* I shall now speak to you about this particular aspect of quality control, and I shall remind you that its main purpose is *re-design* of the quality of your product, and *adjustment* of the plant, *contraction or expansion* of the output of particular products *to meet rationally predicted changes in demand.* These are important require-

ments of trade, domestic and international.

What is consumer research? I have mentioned several times the need for statistical surveys for consumer research. As the importance of the consumer has been left out of the chain of production in many parts of the world where industry needs expansion, I shall append a few remarks on the subject.

Consumer research is an integral part of production. As I said earlier, the terms "good quality" and "quality control" have no meaning except with reference to the consumer's needs. Without consumer research, the product has little chance of being maximally useful, or made in the most economical quantities. In fact, a manufacturing concern can hardly hope to stay in business today without vigorous consumer research.

The main use of consumer research is to feed consumer reactions back into the design of the product, so that management can anticipate changing demands and requirements, and set economical production levels. Consumer research takes the pulse of the consumer's reactions and demands, and seeks explanations for the findings.

Consumer research is not merely selling. Real consumer research, geared to design and production, is an indispensable modern tool for modern problems.

Consumer research is *communication* between the manufacturer and the users and potential users of his product, like this:

manufacturer ⟷ { the user and the non-user

This communication may be carried out today only by sampling procedures. On the basis of this communication, by which the manufacturer discovers how his product performs *in service* and what people think of his product, why some bought it, why

some do not,or would not buy it again, the manufacturer is able to re-design his product, to make it better—better in the sense of meeting the needs of the people who may buy it, and of quality and uniformity best suited to the end-uses, and to the price that people can pay. Consumer research acts as a governor or servo-mechanism which by probing into the future market regulates both the design of the product and the amount of production.

In the olden days, before the industrial era, the tailor, the carpenter, the shoemaker, the milkman, the blacksmith knew his customers by name. He knew whether they were satisfied, and what he should do to improve appreciation for his product. With the expansion of industry, this personal touch was lost. The wholesaler, the jobber, and the retailer have now stepped in, and in effect have set up a barrier between the manufacturer and the ultimate consumer. But sampling, a new science, steps in and *pierces that barrier.* The manufacturer of today, but for sampling, would be out of touch with the people who use his product, or those who might use it.

THE OLD WAY

Manufacturers used to think of manufacturing in three steps, as shown in Fig. 1. Success depended on guess-work—guessing what type and design of product would sell, how much of it to make. In the old way, the three steps of Fig. 2 are completely independent.

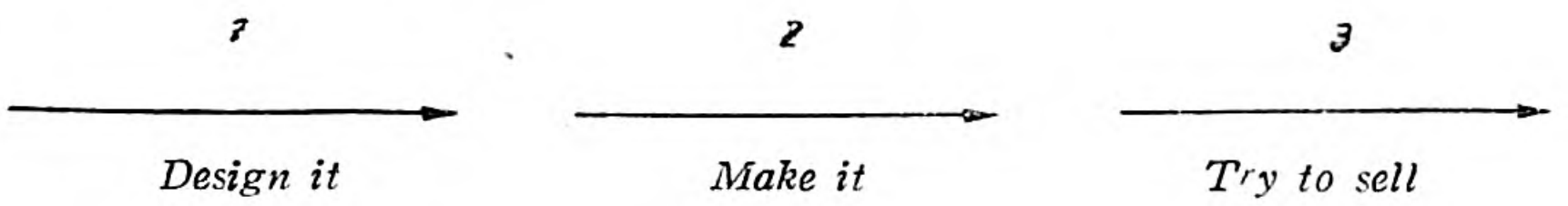


Fig. 2

THE NEW WAY

In the new way, management introduces, through consumer research, a 4th step, and runs through the four steps in a *cycle*, over and over as in Fig. 3, and not in the line of Fig. 2.

1. Design the product (with appropriate tests)
2. Make it, test it in the production line and in the laboratory
3. Put it on the market
4. Test it in service, through market research, find out what the user thinks of it, and why the non-user has not bought it
5. *Re*-design the product, in the light of consumer reactions to quality and price

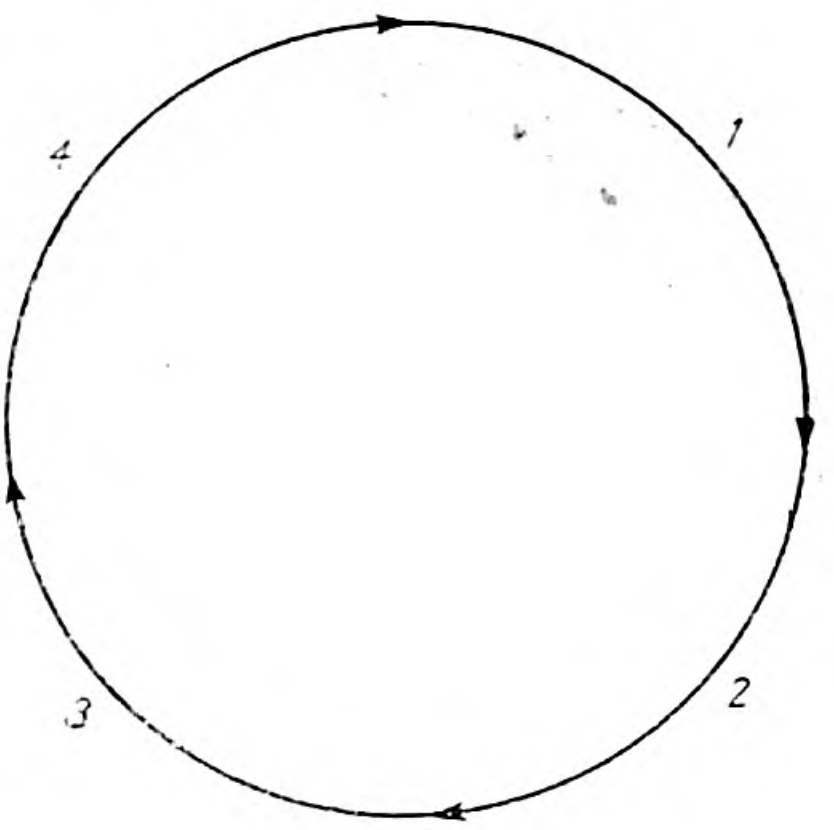


Fig. 3

Continue around and around the cycle.

This 4th step in Fig. 3 was impossible until recently—i. e., it could not be carried out economically or reliably. Modern statistical techniques, such as sampling and design of experiment, combined with the arts of questioning and interviewing, provide information on consumer reactions with economy and reliability. Intelligent manufacturers have always been interested in discovering the needs and the reactions of the user and the potential user, but until recently they had no economical or reliable way of investigating them.

The 4th step, communication between the manufacturer and the user and the potential user, gives the public a chance. It

gives the user a better product, better suited to his needs, and cheaper. *Democracy in industry*, one might say.

A still better way is to begin the manufacturing and marketing of a product on a pilot scale, and to build up its production on a sound economic basis, only as fast as market conditions indicate, re-designing the product from time to time in the light of consumer needs and reactions. The cycle is best taken on a spiral, as in Fig. 4.

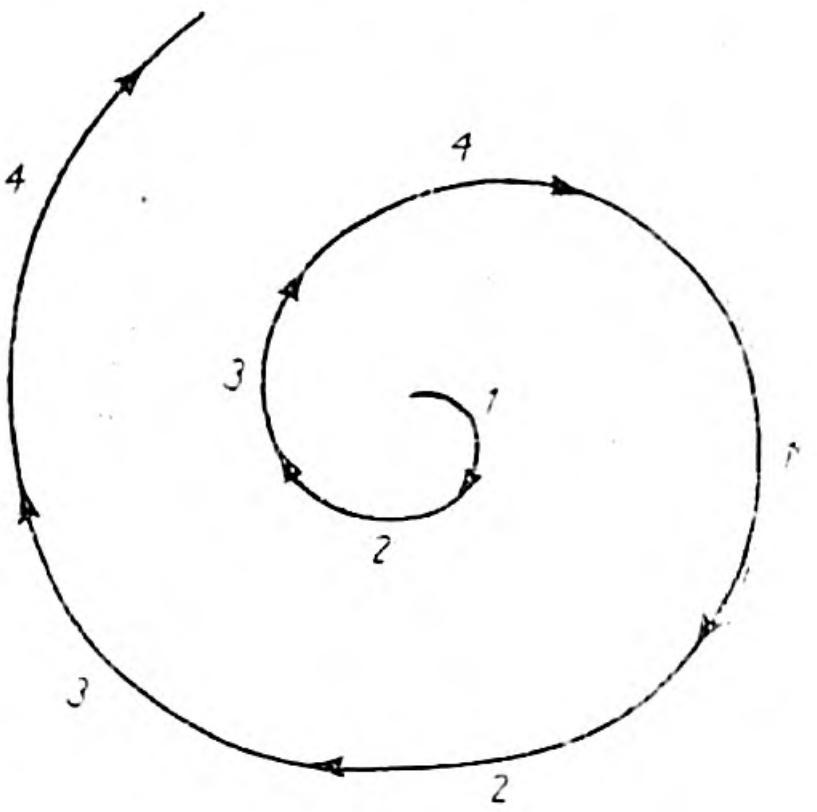


Fig. 4

It is not to be supposed that the first three steps are the same in the figures that display the old and new ways. Consider, for example, design in step 1. Proper design today means not only attention to color, shape, size, hardness, strength, and finish, but attention also to a suitable degree of uniformity —enough, but not so much that the article is priced out of the market. In consumer research, the 4th step, the manufacturer studies the requirements of uniformity, as well as of color, shape, size, hardness, etc. Then, through statistical procedures he achieves uniformity with economy, and his *control charts furnish proof of uniformity.*

Consumer research is a continuous process by which the product is continually improved and modified to meet changing requirements of the consumer. Consumer research, intelligently used, enables the manufacturer to run his factory on an even keel, neither greatly over-producing nor under-producing. He does not let out 300 men one month and try to recover them the next. Consumer research used in this way is a powerful

factor in economical production. Again, it is only one of the indispensable statistical tools of production.

Will you think of some important area in your own manufacturing plant, where some article is being made, shaped, assembled, soldered, or welded? Are the results uniform? Are they 100% successful? Are there any failures? Are these articles inspected? Are any articles rejected afterward in inspection?

Now I ask this question: Do you know how many (what percentage) of these articles were failures or were rejected last week, or on any day last week? Were any records kept? If so, *was any use made of the records*?

Inspection of a product in its various stages of manufacture and assembly is a good idea, but it is not enough. Inspection is not quality control, and quality control is not inspection. Quality control is *action*. My friend Mr. Harold F. Dodge of the Bell Telephone Laboratories once said: "You can not inspect quality into a product." He meant that you must *build* quality: you must make the product so that it has quality in it, if you want quality. Quality is not built by making a great number of articles, hoping that some of them will be good, and then sorting out the bad ones. Your company or anyone else's, whether it is in Tokyo or Chicago, will go bankrupt if it attempts to operate in that way today.

Here you will learn something about the use of statistical techniques for cutting the costs of manufacturing, and in improving the quality and uniformity of your product. As I said earlier, you can not learn the entire subject in 8 days. You will learn some elementary principles. I hope that you will continue your statistical studies for many years. If you are going to succeed today as engineers, executives, or economists, you must know and use a lot of statistical theory.

If I succeed in inspiring you to continue your statistical studies for many years to come, then my efforts in this course will be successful, and I shall be happy.

The lectures that follow are patterned after the 8-day courses that were so successful in America, beginning in 1942, which it was my pleasure to introduce. It was numerous 8-day series of lectures throughout the country, and the nucleus of discussion groups which the instructors planted in many large manufacturing centres, which really started the use of the statistical control of quality in America and laid the foundation for the American Society for Quality Control.

LECTURE 1

CONTROLLED AND UNCONTROLLED VARIABILITY

Variability is a rule of nature. Repetitions of any procedure will produce variable results. The diameters of any lot of manufactured parts will not all be alike: they will vary. The causes may or may not be apparent. Of 20 workers, the production of some will be below the average of the 20 during some particular week. and the production of others will be above the average. Yet the workers who fall above the average one week may fall below the average next week. You plant beans: they do not all come up at the same hour. One comes through, then another, etc.. and finally the last straggler.

Why does variability exist? We can not know all the answers, but it is useful in science and manufacturing to recognize two kinds of variability—

Controlled variability

> It will *not* be profitable to try to *determine* the cause of individual variations when the variability is controlled.

Uncontrolled variability

> It will be profitable to try to *determine and remove* the cause of uncontrolled variability.

One form of "TROUBLE" is uncontrolled variability, and the cause should be found and removed. With uncontrolled variability there will occasionally be points falling above or below the indicated control limits.

The *Shewhart Control Chart* is a statistical tool that aids the

engineer or foreman to separate these two kinds of variability.

Why separate them? For economy in production and inspection. In production, we wish to guard against two kinds of mistakes, both of which are costly.

(a) Hunting for trouble and making adjustments when no trouble exists.

(b) Failing to hunt for trouble and failing to take action when trouble does exist.

It is possible to avoid either mistake completely *but not both.*

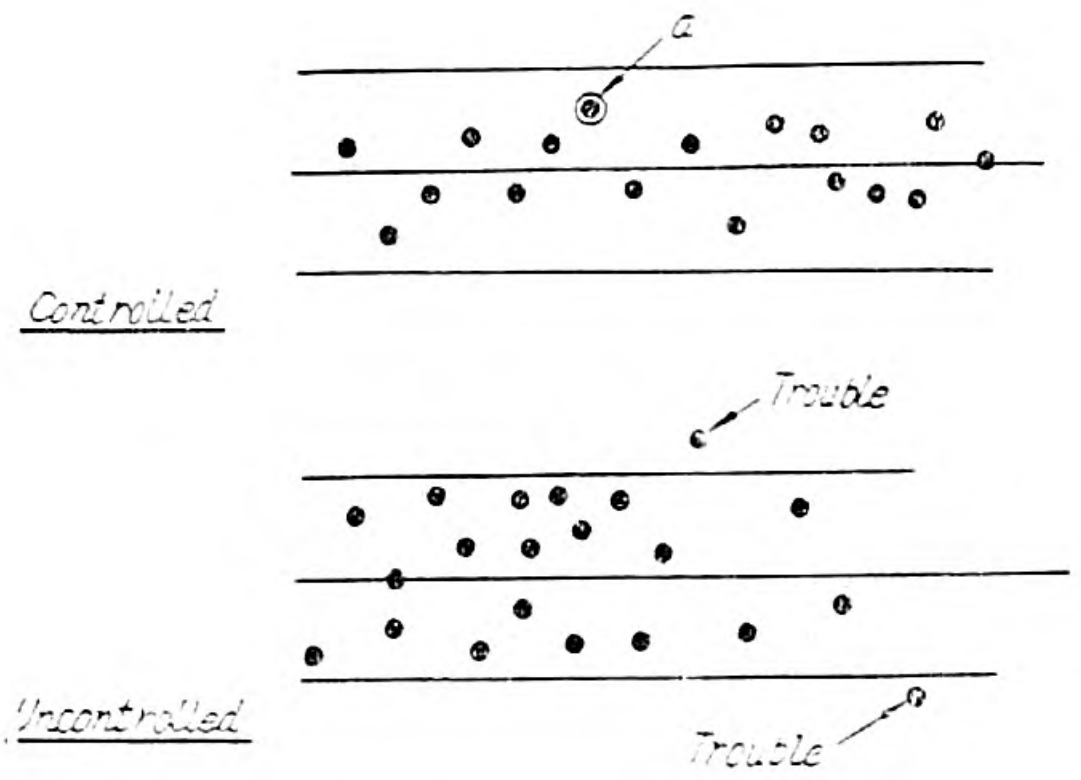


Fig. 5. The Shewhart Control Chart

Before we go ahead I wish you to think for a few minutes. Now look at Fig. 5 and point a which is above the average. Will you make an adjustment when you see this point? The answer is no. If you make an adjustment of the machine at this point you may throw the whole process out of control. Now this is important. If you can understand this you are as good as half through this course.

With the aid of the control chart we *minimize the net cost of both kinds of mistakes.* We do not hunt too often for trouble, nor fail too often to hunt for trouble.

Now, I am going to ask you a question. I want somebody to answer.

In 1942 I was with the personnel director of a large corporation in Detroit, *Fortunately* I cannot recall the name of the company. Now this personnel director told me that at his company the management notified each week any worker who was producing below the average. He wanted to know what I thought about the system; and he was very proud that they had a way of knowing just how every worker—23,000 of them—was doing every week. I told him: "I don't think your company is strong enough—no company in America is strong enough—to be able to tell anyone that he was producing below the average, and continue in business for half a year." I want you to tell me which of us — he or I—was correct; and tell me why I told you about this.

Student: It is necessary first to determine the cause of individual variability before passing it on to your workers.

Deming: I think that is correct. That is my point of view. Actually it makes no difference whether it was being done every week or once in six months. His method was wrong. It would have been quite all right if he had got down to determining which of his workers were above or below control and had taken appropriate action in regard to them. Now, what would you have done in regard to those whose production was above control? What would you do with those who were below?

You would first investigate the causes and then take action. With those coming below control you might take them out and put them to other kinds of work. One of the causes of low production might be too much education. It might happen that the man is thinking too much about quality control to be able to produce enough. In that case he should be put to work on quality control.

Now I said that the mistakes of hunting for trouble and making adjustments when no trouble exists, and that of failing to hunt for trouble and not taking action when trouble does exist are both costly. I also said it is possible to avoid either mistake completely, but not both.

Later, I shall ask you to tell me how you would avoid either one of the mistakes completely.

With the aid of quality control we can minimize the net cost of both kinds of mistakes. With the quality control chart we do not hunt for trouble too often, nor do we fail too often to hunt for trouble.

Here study Sections 4, 5, 6, 7, 8, 9 of your "Guide for Quality Control" (Z1. 1).

Note that the use of the control chart will enable you to cut costs chiefly by (a) attaining better and more uniform production methods; and (b) decreasing the amount of inspection that is necessary to produce the desired quality.

Note also that the control chart by itself will not improve quality or uniformity. You must take *action* on the process if you wish to achieve any improvement. That is, when a point goes out of conttol, you must then try to *find* the cause, and *remove* it. The control chart only indicates when you should look for trouble and do something about removing it. It is *action*, not the control chart, that will achieve results.

Now we come to the question: "Can you avoid either one of the mistakes (a) and (b) completely?"

Mistake (a)	0	100	x
Mistake (b)	100	0	y
			minimum net loss

Student: The mistake can be eliminated by obtaining accurate specifications from the purchaser and also by adequate

checks so that all the points fall within the control limits.

Deming: That is good, but has someone else another answer?

Student: You can not avoid both mistakes completely. If you avoid one of them completely, you will commit the other one on every possible occasion.

Deming: That answer is perfect. At least that was the answer that I wanted. To make mistake (a) on every possible occasion, you never make an adjustment. You can avoid mistake (a) by never hunting for trouble. Never make any adjustment or change. Well, what happens when you do this? Then you commit mistake (b) on every occasion that is possible.

Now, the converse is true. If we avoid mistake (b) all the time we will be committing mistake (a) every time that it is possible. What is the aim of quality control?

Student: To seek the most economical solution.

Deming: That is an excellent answer. We want a little of (a) and a little of (b) so that we minimize the net loss resulting from both mistakes. How do we achieve this and by what tool?

Student: By utilizing statistical methods.

Deming: By statistical methods you probably mean to enlist the aid of control charts. Your answer is good. What does the control chart do? It minimizes the net loss from both mistakes. To do so, you separate the variability, distinguishing between controlled and uncontrolled variabilities. You do this by the control chart. Then we see two things. We minimize the net loss of the mistakes (a) and (b) and separate—that is distinguish between—controlled and uncontrolled variabilities. Later we shall see many examples of controlled and uncontrolled variabilities.

Now, I should like to ask one of you to bring me inspection

or test data of some sort—25 or more samples. Then we shall make a control chart out of the material and see what the process of control actually amounts to. This we shall do later; and now we shall take up some of the important ideas in your text-book, "Guide for Quality Control" Z1.1.

Section 4. First of all the "control limits" are not "specification limits." The control limits are used as a basis for judging the significance of the quality variations from sample to sample, or from lot to lot, or from time to time.

Section 5. What is given in this section is a repetition of what I have told you.

When an *assignable cause* (of trouble) is indicated you have uncontrolled variability and it will be profitable to try to determine and remove the causes of it.

Section 6. "When a state of statistical control is reached, no higher degree of uniformity in quality can be attained with the production process in use." When I have reached the state shown in Fig. 6 how can I attain greater uniformity?

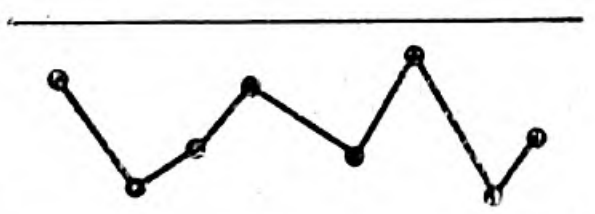


Fig. 6

Student: Revise the production process.

Deming: Will you tell me more clearly what you mean by *revising the production process*? Do you mean to eliminate more causes of uncontrolled variability? If you attempt to do so, either by means of the control chart or by other means you would be creating further causes of variability.

Student: There would be two methods. First, to have a revision of the raw material specifications and those of the manufacturing process. The second would be to change the control limits.

Deming: How would you change the control limits? Control limits are determined by the process. Change of specifications

will not help. What is needed is a basic change in the manufacturing process. What would be a *basic change* in the process?

Student: Change the equipment entirely.

Deming: Use of better and more expensive raw materials might be less costly to you.

Student: I would change the operating standards.

Deming: That is correct. However a change in operating standards would mean a completely new training program. A basic change in the methods of operation sounds effective. But what I am trying to place before you is that when you have reached the state of statistical control it is something very basic. You must have basic training to achieve good uniformity. The really important idea is that the basic change entailed may be very expensive. If may be too expensive, and new mistakes may result in an extremely costly procedure. Various raw materials may not be economical—that is, they may not be profitable to use.

Therefore when you have reached the state of statistical control you must be satisfied. You must either be satisfied with this degree of uniformity in quality or else be prepared to undergo basic changes at considerable expense—perhaps unprofitable expense. Another important thing that you should remember is that, generally speaking, a state of control signifies a definite process.

Now, I wonder now many of you people know what golf is? It is a game which I played once and lost the ball quickly. Well, it is a game; and as I know it the aim is to make as low a score as possible. I do not know much more about the game.

However there is a friend of mine who plays a lot of golf. Incidently his name is *Russel Passano.* Now this man took his game of golf pretty seriously, so he kept track of his playing—

his performance at the game—over a period of several months. In order to do so he took five successive scores and called it a *sample.* In other words his sample was a sample of five units. If, for instance, his scores had been: 100; 120; 90; 110; 100, then the *average* for the five would come to 104, while the difference between the highest and lowest of these five scores would be 30. This difference is known as the *range.*

He played a great deal of golf, and so after playing 125 games he was able to plot 25 points on his control charts for average and range.

Now, incidentally, this Mr. Passano was the first man to do quality control work in the steel industry. He started way back before 1930 with the Bethlehem Steel Company, and since that company is one of the biggest in America Mr. Passano had plenty of opportunity.

Getting back to Mr. Passano's game of golf and the control chart that he plotted for it, he plotted 25 points, looked at the results and was not satisfied. He decided that he had still lots more to learn; that he could improve his playing if he took lessons. So he took a dozen or so lessons; and after that he played some more golf, and again plotted his scores.

Now, what do you think happened? Can you expect that Mr. Passano's game of golf improved?

Suppose we extend the control limits of Mr. Passano's game beyond the point where he took lessons. Where would the plotted points fall? Now, here is another control chart plotted for a bad golfer whose technique was not under control (Fig. 7) What would happen after this player took lessons?

Student: In the case of Mr. Passano, the points would continue to fall within the original control limits, but with the poor player there should be some improvement.

Deming: You are absolutely right. Mr. Passano, after

taking lessons again plotted 25 points from his samples and found them all falling within the control limits that he first computed and set. In other words, the twelve or more lessons he took may have given him some pleasure but they did him no good—they were a total economic loss to him in time and money. With the poor player, however, it should be an entirely different story.

Since the bad player's game was out of control in the first place, the lessons should reduce his scores and decrease the amount of variability from sample to sample.

In the case of Mr. Passano, no apparent improvement took place. Now, lessons are taken in order to improve your skill —to enable you to get better control over your playing. With Mr. Passano, there was no improvement because he had as much control as he could possibly acquire in the first place.

This is a good illustration of an important application of the control chart. In industry an operator of a machine can develop his technique to the point of statistical control and no further. Suppose you have practised using a piano or a typewriter. You might go on for twenty years; you are developing your technique, and it may come to a state of statistical control which can be shown by the number of errors you still make. When you first started you surely showed lack of control; and you improved as you went along. Now, with an operator of a machine or a soldering iron, it amounts to the same thing. And the earlier the training is started to assist the technique in the right way, the better.

As I mentioned a short while ago, Mr. Passano's game showed no apparent improvement. In other words his lessons were a total loss — just a waste of time, unless Mr. Passano actually enjoyed them. You see he had a controlled process, and since the lessons did not introduce a fundamental change, he gained nothing. With the bad player matters are quite different. He

has no controlled process — he is still teachable. And after taking lessons he can expect better and more uniform scores. This is very important. When you have a controlled process you have as much uniformity as you can get from that process; but with an uncontrolled process you can do anything.

We have a control chart here. (Fig. 8) Remember it is for $\bar{X}$. Now, instead of golf scores, let us assume that each of the 50 points stands for the average value of five articles. Then we shall have 250 individual values. Now, suppose 10% of these individual values fall below the specification unit. What should I do to get rid of the defective articles? If the whole $\bar{X}$ level must be raised, how would you do it?

A fundamental or basic change must be made in the pro-

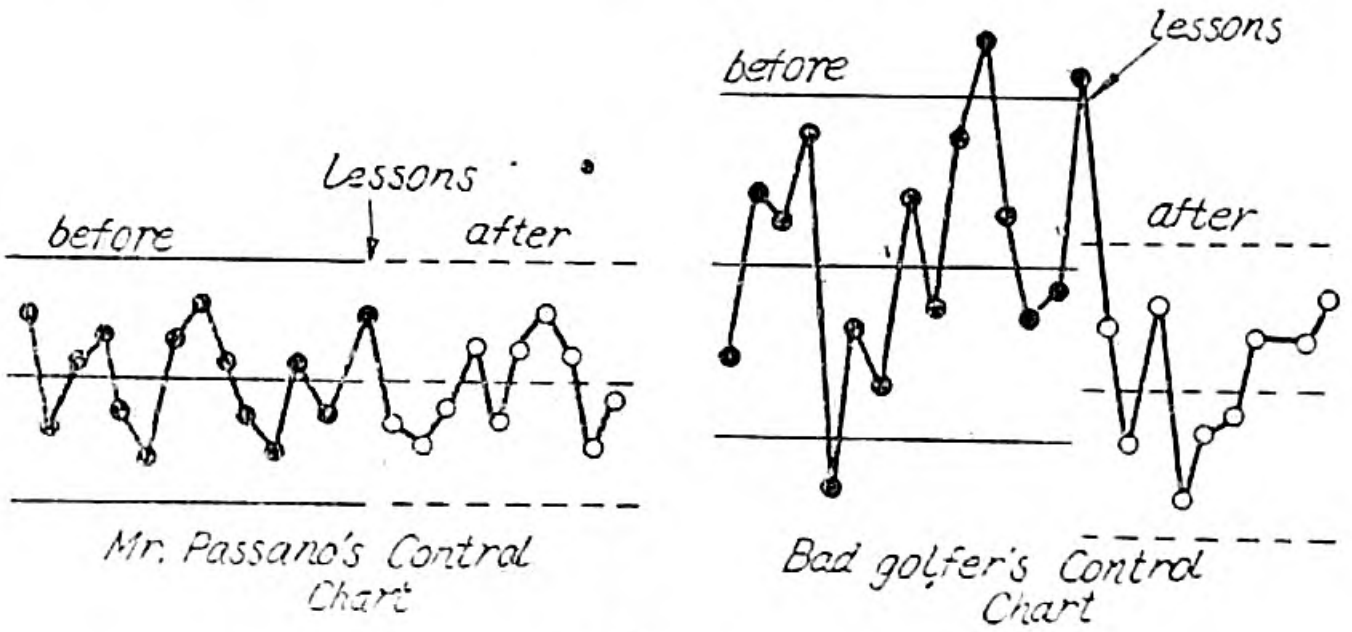


Fig. 7

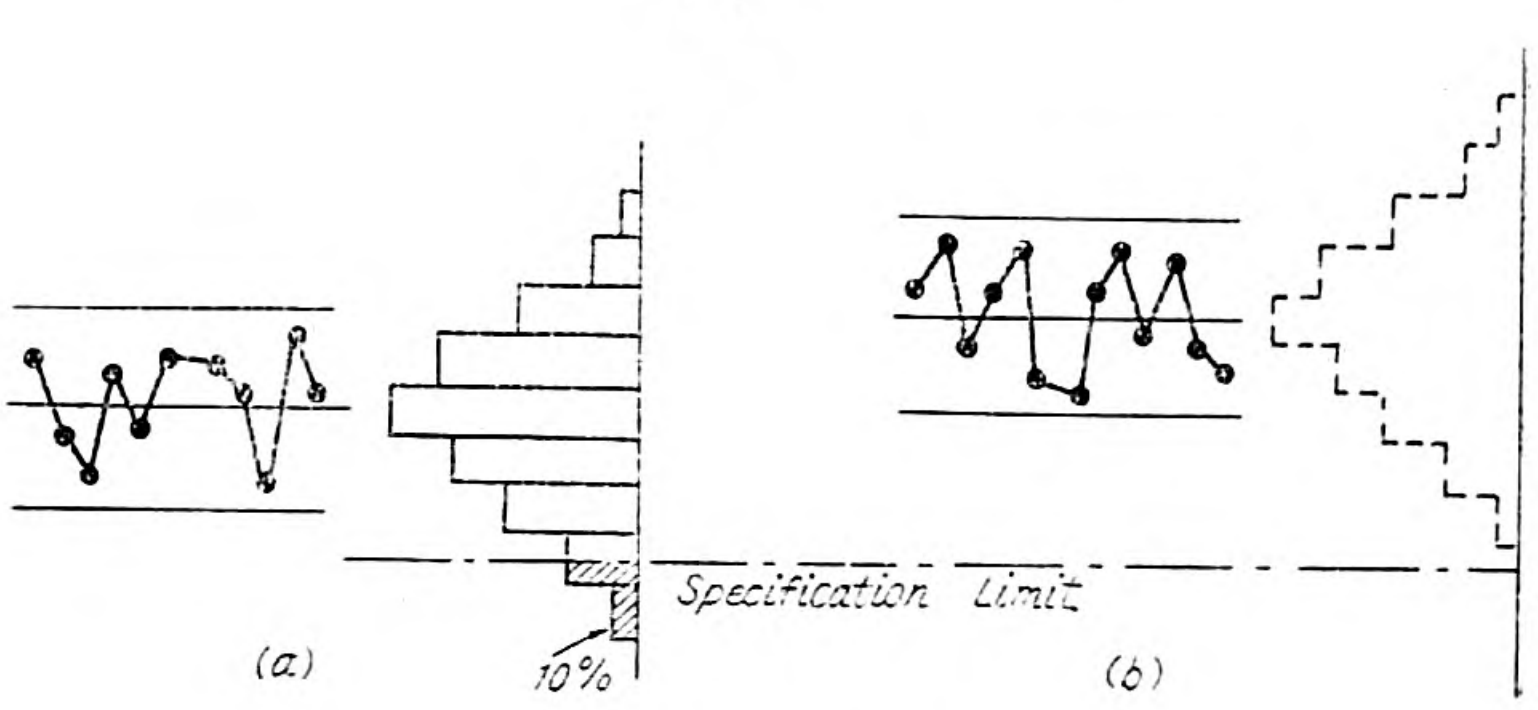


Fig. 8

cess to raise the level so that quality is controlled sufficiently above the specification level. We may require a fundamental change in the process.

But it may be expensive, it may not be economical to raise that level because it may require new machines, new tools, retraining of workers. Can anyone think of another way of eliminating the defectives?

Student: Lower the specifications.

Deming: Well, if we don't do that? The specification limits are given to us. In what other way could we make production meet the specifications?

Student: It may not be economical, but reject the defectives.

Deming: Correct. That is the only way. If 90% are *good* and 10% *bad*, take out the bad ones by inspection. This is one way to meet a specification, but it is expensive. How do you find the defective 10% of the articles?

Student: By individual inspection of the whole work.

Deming: That is correct. I am only pointing out to you that it isn't easy. In order to eliminate the defective 10% how many or what percentage should be inspected?

Student: Apply statistical methods to find the correct percentage.

Deming: No. The only way by which every bad piece can be found is by 100% inspection. You can try. That is all. Individual inspection of every piece is not a perfect way of finding all the defectives. If you want to find all of the bad pieces—all the 10% bad — so that you can keep the process you have, then you must be sure that you always find the defective 10% of the output. But the inspector can not always find all the bad ones.

What can you do? There are some people who will do 200%

inspection. If that is not enough, 300%. Now, this is actually done; and no doubt some of you have experienced that already. But it must be kept in mind that this is an expensive procedure. These are troubles that all production engineers should bear in mind.

Now, here is another method that has been suggested. It was recommended that the process be changed and the level raised so that the distribution of individual values would always come above the specification limit.

When you have a state of statistical control and are making 10% defectives, and are required to deliver to your purchaser only perfect articles and no defectives, you must do one of these two things: either inspect perfectly or raise the level. Remember, the adoption of either solution costs money. You must make the choice before you give instruction to your engineers. You know the cost of inspection and you can estimate how much it will cost to change the process to a higher level. You will naturally choose the cheaper method. However if the process is not controlled, you have another problem which we shall discuss later.

In *Section* 7 of Z1.1 we come to some very important ideas.

(*b*) With control, data from samples of the product have the greatest possible reliability as a basis for judging its quality. Sampling and testing, and hence the cost of inspection, can be reduced to a minimum. As a result, it often follows that sampling inspection is adequate, both for the manufacturer and the purchaser.

The reliability of sampling results is particularly important when assurance regarding the quality of an entire quantity of similar articles or material must be based on the inspection of only a limited number of articles or test specimens. This will be the situation when 100 per cent inspection is impracticable

or impossible, as happens when the inspection test of an article is destructive or when tests are made to measure the physical or chemical properties of materials, bulk products, or the like.

Even when 100 per cent inspection is practicable, but where it is satisfactory to use sampling inspection in place of 100 per cent inspection (as when 100 per cent conformance to specification requirements is not essential), the possible savings in the cost of inspection are greatest when the quality is controlled and the level of control is a satisfactory one.

I would like to tell you what this means. Suppose you have a controlled process of a certain level, and you produce a certain distribution which comes well above the specification. Then you don't need 100 % inspection. Why do you need any inspection at all? The lower limit of distribution is well above the specification. Why inspect at all? What is the answer?

Student: Sampling and testing will still be necessary.

Deming: For what purpose?

Student: To determine whether a state of control exists.

Deming: Correct. That is a perfect answer. The idea is very important. What do you have to do to maintain statistical control at the right level? You don't need 100 % inspection. All you need do is to inspect your product to make sure that the state of control exists. When you are in a state of control at the desired level, you are producing the same distribution hour after hour, day after day. And the lower limit of the distribution is above the specification limit. You may thus reduce the cost of inspection, perhaps to 1 % or even half per cent. Then inspect only to make sure that your state of statistical control is maintained.

(*c*) With control, the percentage of product whose quality will lie within any pair of limits may be predicted with the highest degree of assurance.

(d) This paragraph is very important, but I shall skip it just now and come back to it later.

(e) With control, acceptance of a product by the purchaser may often safely be based on the manufacturer's evidence of control, rather than on results of the purchaser's own sampling, shipment by shipment. This may be accomplished if continuing control chart records are by agreement made available by the manufacturer to the purchaser, who will then need to test only occasional samples as checks on the quality records furnished by the manufacturer.

Suppose you are buying products from another manufacturer, or you are in one department of a plant, and your raw material comes from another department of the same plant. If the product you are buying or receiving is produced under control then, you can arrange to be furnished with the control records. A lot of business is done in this way. Today, when you draw up a contract for purchase of a product you can arrange so that the manufacturer is required to furnish you not only with the products ordered, but with copies, blue prints, of the control charts covering the manufacture of the lot you receive. If you can be assured that the process was in a state of statistical control the cost of inspection can be reduced to almost zero. The contract can be drawn up so that if the manufacture loses control, he pays the cost of inspection to eliminate the defectives.

Section 8. Establishment of Control Limits

These limits are determined from calculation and from data:

(a) Use of manufacturing data, past and present, of a certain product for determination of its control limits; and

(b) Use of mathematical formulas derived from statistical theory concerning quality control, and known from past experience to give reliable and practical results.

Section 9. Principal Uses of the Control Chart

For the field under consideration here, the control chart has two principal uses—assisting judgment with regard to whether a state of control exists (section 10), and attaining control of quality during production (section 11).

The ultimate and most important purpose of the control chart is to provide a definite operational procedure for controlling quality in the manufacturing plant. However, before setting up a system of control of future operations, it is usually necessary to get a picture of the history of the quality of the product and in consideration of this information, to decide what tentative control limits should initially be placed on the chart. Therefore, in practice, the first purpose for which the crntrol chart is used is often that of analyzing the quality records of the past.

The first use you may make of a control chart is to discover whether a certain operation in your factory is under control. Now if you find that statistical control does not exist—it simply doesn't come to you as a gift—then you must achieve it by eliminating one cause of trouble after another. It may take months and months of effort.

LECTURE 2

SOME ELEMENTARY IDEAS REGARDING THE CONTROL CHART

Now we shall have to turn again to the "Guide for Quality Control" and study the following sections as we proceed.

Section 10. in Z1.1. Use of the control chart for judging whether control exists.

Section 11. in Z1.1. Use of the control chart for attaining control of quality.

Section 12. in Z1.1. Cooperation between the technical man and the statistician.

Now, suppose you take a look at the control chart on page 9. In the upper panel we have the *original data.* They are individual values. The points represent inspection records for individual articles. You have five successive articles for one sample (subgroup). In the lower panel of Fig. 2 you have the points representing the averages for each *subgroup* (sample). Ordinarily we don't show the individual values. They are plotted here to make the construction of the control chart clear. 25 samples are enough for a control chart, but you can start with less if you wish. Here we have 25 values on the $\bar{X}$ chart. The sample (subgroup) numbers are on the horizontal scale. You will note that samples 17 and 21 are *out of control.* The chart shows just where the *trouble occurred.* Each sample was made by certain people, at a certain time, with certain material. However the control chart does not find the trouble for you. You have to find it. You have to go a long way to find the actual difficulty : when it occurred, with what people, what machine and what material. You need not do much more. You find the trouble, then you *must eliminate* it.

This is a control chart which was plotted in order to get started. It shows lack of control at two points. Now, if the

chart is six months old it will be very difficult for you to find the troubles at points 17 and 21. But if the chart was made from yesterday's or this morning's records you can perhaps find the trouble soon. It is much easier to find trouble if it occurred this morning or yesterday rather than 6 months ago. Therefore control charts should be made *promptly*. Speed is important when you have to eliminate the cause of trouble. When you have eliminated the trouble, you collect more data and plot a new chart, or just put new points on the chart and see what you have.

Section 11. Use of the Control Chart for Attaining Control of Quality.

I think this is a very important section. Please read it carefully. You use the control chart at the spot in the production process where you think there is trouble. Sample 1 is from operator 1, sample 2 is from operator 2, and so on. Now you have 25 points representing 25 workers. The points can represent anything—production (output) or error rates for five successive days for each operator. If any operator is out of control he is different from the others. Assuming we are dealing with output, then operators 17 and 21 have high production rates; not ony high, but statistically high. What would you do about such high production?

Student: They have something that would be worthwhile to study.

Deming: Yes. Make a study of their methods and you might get to know something profitable.

Now if you are plotting the error rate—number of mistakes—and you find some workers out of control below the lower control limit. What would you do about them?

Student: In this case something should be done to try to get them back to ordinary standards.

Deming: Yes; these workers are having some special difficulties. They are somehow not suited to the job, perhaps some re-training will help: perhaps it will not. They may have too much education, or too little. Perhaps they should be moved into other work, at some other point in production. Study is required of every case. Remember that there is a huge investment in every worker, and that human workers are not like physical materials. Try to find a happy solution.

Note that real quality control is *not* something that you "install" in a plant, like installing a machine. Quality control is *built*. It grows, like a tree. The ideas and principles must take root. They must proceed methodically with a definite aim. Time will be required for you to build quality control. And I repeat, you can only be successful in quality control if you control the right process. So proceed carefully. Keep in mind all 4 sectors of Fig. 3 in the introduction.

LECTURE 3

PLOTTING A CONTROL CHART

CONTROL CHART METHOD FOR JUDGING WHETHER CONTROL EXISTS

Here we have some actual data for an $\bar{X}$ and R chart. The characteristics are: *height of coal briquette in millimeters.*

Now plot your charts. If you have ruled paper, use it. If you have no ruled paper, scale off some plain paper as accurately as you can. A common fault of beginners is to use too wide a scale — about 25 millimeters between the control limits will be enough.

Each sample was taken from a day's production. There

The control limits for $\bar{X}$ and R are:

$\bar{X}$ Chart ·····	Upper Control Limit,	33.03
	Central Line,	32.49
	Lower Control Limit,	31.96
R Chart ·····	Upper Control Limit,	1.96
	Central Line,	0.925
	Lower Control Limit,	0

Fig. 9. $\bar{X}$-R Chart of Coal Briquette

Table. 1

For $\bar{X}$ and R–chart

Name of Co.

Date 7. 12

Article Coal Briquette Lot number

Operation Finish Name

Type of measurement Height of coal briquette in mm

Remarks Each sample is from 1 day's out put 21 days

Sample number						$\bar{X}$	R	σ
1	32.20	32.60	33.30	32.40	32.20	32.92	1.30	0.495
2	32.10	31.95	32.15	31.10	32.70	31.99	0.55	0.67
3	31.85	31.15	31.90	31.40	31.20	31.50	0.75	0.31
4	31.30	32.00	31.75	31.50	32.00	31.71	0.70	0.28
5	32.35	32.40	32.60	31.85	32.45	32.33	0.75	0.30
6	31.80	32.90	32.45	31.95	31.60	32.14	1.30	0.67
7	32.75	32.60	33.85	33.00	33.20	33.08	1.25	0.55
8	32.85	32.90	33.30	32.40	33.85	33.06	1.45	0.56
9	33.30	33.20	33.20	33.20	32.80	33.94	1.10	0.41
10	33.15	33.40	33.40	32.40	32.40	33.99	1.10	0.46
11	32.20	32.45	32.90	33.20	32.75	32.72	1.00	
12	32.90	32.40	32.85	33.90	32.90	32.79	0.50	
13	32.95	32.90	32.70	33.00	33.70	33.05	1.00	
14	32.75	33.30	32.90	32.90	33.20	32.89	1.00	
15	32.15	32.75	32.70	32.15	32.80	32.61	1.00	
16	33.00	33.10	32.70	32.60	32.45	32.81	0.50	
17	32.60	32.45	32.80	32.45	32.45	32.55	0.35	
18	31.45	31.85	31.90	31.45	32.00	31.75	0.55	
19	32.75	32.75	32.70	32.80	32.95	32.79	0.25	
20	31.00	31.50	31.50	32.00	31.45	31.67	0.60	
21	32.75	31.70	31.85	32.45	31.90	32.15	1.50	
22								
23								
24								
25								
Total								
Average						$\bar{\bar{X}}$ = 32.493	$\bar{R}$ = 0.925	$\bar{\sigma}$ =

On the $\bar{X}$–chart: Central line, $\bar{\bar{X}}$ = 32.493 $A_2\bar{R}$ =

UCL, $\bar{\bar{X}} + A_2\bar{R}$ = 32.49 + .577 × .925 = 33.03

LCL, $\bar{\bar{X}} - A_2\bar{R}$ = 32.49 − .577 × 925 = 31.96

On the R–chart: Central line, $\bar{R}$ = 0.925

UCL, $D_4\bar{R}$ = 2.114 × .925 = 1.96

LCL, $D_3\bar{R}$ = 0

$\bar{\bar{X}} = 32.493$ $\bar{R} = 0{,}925$

See Table for $n = 5$

$A_2 = 0.577$ $D_4 = 2.144$ $D_3 = 0$

are 24 days on the chart. Do the plotted points show statistical control? Do you find any point out of control?

Student: 8 points are out of control.

Deming: What would say the chart indicates? Does it indicate stable conditions?

Student: From the 7th day to the 13th day most points fall near the control limit. During this period there must have been some misadjustment.

Deming: Well, that is a clever answer. It may have been misadjustment of equipment. But there is something more than that. From point 7 to point 17, actually 11 points altogether are above the central line. 7 points near the upper control limit and 3 points out of control indicate that there was something unusual between the 7th and 17th days. There is something definitely different from the other days. What could the trouble be? Equipment? Or are there other possibilities? Weather? Condition of the raw material? The coal from which the briquettes were made?

In this case you should put in the central line between the limits (indicating the average of the averages, $\bar{X}$). From point 7 to point 17 all the points are above the central line.

On these 8 days is there anything unusual about the range? I don't see anything unusual. The range is controlled. It is the variability of individual values from day to day that shows something unusual. Now I am going to ask a question. Why is it necessary to compute the ranges? Suppose you plot an X-chart. Why must you compute the range?

Student: The range must be computed so that the control limits can be set.

Deming: Perfect. The *average range* is $\bar{R}$. The distance between the control limits of $\bar{X}$ is determined by the average range. The X-chart is thus in some way a comparison between

the variabilities of the $\overline{X}$ points with the variabilities among the individual values. On the $\overline{X}$-chart you look at the variabilities from sample to sample. You compare this variability with the average range. What then is range? The distance between the highest and lowest points. Any questions?

Question: In case there is no stability, how is $\overline{X}$ looked upon?

Deming: I have several good answers for that. It must be that there is a condition that varies from day to day. With the briquettes for instance, it might be the size of the coal that comes into the plant from day to day. What can be done about that? What would you suggest be done in order to bring about control?

You were able to discover variability with the control chart. Without the chart you might never have suspected that such day to day variability existed. Now suppose the engineer in charge decides that the variability indicated by the $\overline{X}$-chart arises from the difference in the size of coal coming into the plant day by day. What can he do about it? Remember that the control chart by itself is not going to change anything. It's you or the engineer who must do something. You have found indication of trouble. What would you suggest?

Student: I would sort out and classify the raw material, then afterwards I would draw up another control chart.

Deming: Would you arrange to set a level of quality for the raw material?

Student: I would first classify the sizes of coal, then with the process in control I would be able to find what type of coal meets the specifications.

Deming: I am not sure about that. Why not use the control chart you already have? There is an indication in that control chart. From the 7th to the 17th day there should be

distinct records of where the coal came from. See what kind and size, and compare with the coal used on the other days. I don't disagree with your suggestion, but I think that the answer is already in the control chart.

Student: According to the control chart all the points from 7 to 17 are above the average. The other points are below average. Therefore it seems that all the above-average coal came from one source while the other coal came from another source.

Deming: You look at the records and see where the coal came from. The control chart is here for a purpose. Direct your attention to variabilities from day to day. Don't assume something. The points between 7 to 17 by themselves would give a beautiful chart showing excellent control. But what we are trying to do is to attain uniformity in the $\bar{X}$. So to check on the conditions between the 7th and 17th days, you look and see where the coal came from. If the coal was always the same, then we are wrong. From the day-to-day differences we must find what is wrong. The control chart is a *tool*. It has directed your attention to the coal. If the cause of the variability is the differences in the size of coal, what would you do?

Student: Adjust the quality of the coal or adjust the machinery for certain types of coal.

Deming: What else?

Student: I would try to get uniform raw material or try to get uniform quality by mixing coals from different sources.

Deming: Although I am not an expert in coal, I think this blending of materials to obtain uniformity is a good idea.

LECTURE 4

SOME EXPERIMENTS WITH A CONTROLLABLE PROCESS

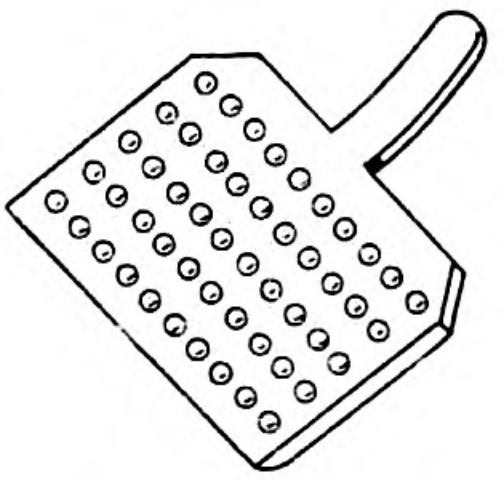
Taking up *Section* 15 (p. 17, Zl. 3), we come to the control limits for the control chart for p. Now I have here an apparatus that is able to produce controlled values of p. So please take your paper named "Chart for p" and we shall collect some actual data. Also I should like someone to bring in from his plant some data on fraction defectives so that we could have a working example.

Now, the apparatus that I have here consists of a box which contains *white* and *red* beads, each about 5 millimeters in diameter, and a paddle or scoop which has 50 holes into which the beads will fit, 5 lengthwise and 10 sideways. We shall scoop out the beads, call the white ones *good* and the red ones *defectives* or off-color. The action of scooping out the beads is the production process, thus.

(demonstrates how it is done)

1st	scoop	11	red	5th	scoop	5	red
2nd	〃	12	〃	6th	〃	5	〃
3rd	〃	10	〃	7th	〃	12	〃
4th	〃	8	〃				

Why is it that the number of red beads, the defectives, varies? The same beads, in the same box, scooped out by the same paddle worked by the same operator. Why does it vary? Does anybody have an answer? Why don't I get the same number each time?



Student: There is no special reason at all.

Deming: That is correct, but I'd like to say just a little

more. I do not do the scooping really in exactly the same way. So the beads come out from a different set of conditions. Now why is it that I do not execute the same motion from the same position every time? Why can't I do it? Does somebody have an answer?

Student: It is impossible to attain uniformity in the mixing of the beads.

Deming: But I think it is a uniform mixture. If you say so it may be impossible. It is impossible because there is a great number of beads. If the number were small enough it might be possible to replace them each time in the same position.

No. The answer is that it is impossible for me to execute the same motion in the same way. The number of defective articles is as near uniformity as I can possibly get it.

Student: Under the present conditions it will be impossible for you to obtain better uniformity.

Deming: I think that is a very good answer. Well let us take our paper. We will draw 25 samples and plot a chart for them (Table 2).

It is not an easy thing to devise a controlled process even in a laboratory where things are well under control. I am not at all sure that what we are doing now is a controlled process: the box is too narrow and I cannot scoop with the paddle in just the way I wish.

[Draws 25 samples and has the number of defectives noted down for each]

Now let us plot a chart. First of all we compute the control limits. I will let three people give me the answer for $\bar{p}$ and take the *average*. Suppose I take 0.2 (*cf.* Table 2, footnote). Now the chart. Again do not make your scale too big. One centimeter above and below the central line is enough. Re-

Table 2.

For *p*—chart

Name of Co.

Date 7 12

Article Beads

Lot number

Operation

Name

Type of defective Red beads

Remarks

Sample number	n	Number defective, np	Fraction defective, p
1	50	14	.28
2	50	11	.22
3	50	11	.22
4	50	9	.18
5	50	13	.26
6	50	9	.18
7	50	12	.24
8	50	11	.22
9	50	5	.10
10	50	13	.26
11	50	9	.18
12	50	15	.30
13	50	13	.26
14	50	12	.24
15	50	9	.18
16	50	10	.20
17	50	13	.26
18	50	14	.28
19	50	4	.08
20	50	5	.10
21	50	14	.28
22	50	5	.10
23	50	2	.04
24	50	9	.18
25	50	8	.16
Total			
Average	$\bar{n} = 50$	×××	$\bar{p} = .200$

Central line, $\bar{p} = .200$

UCL, $\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/\bar{n}} = .200 + 3\sqrt{.2 \times .8/50} = .2 + .17 = .37$

LCL, $\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/\bar{n}} =$ $.2 - .17 = .03$

member the control chart is not something with which to measure size. It only shows general conditions or trends. So we do not need a wide chart.

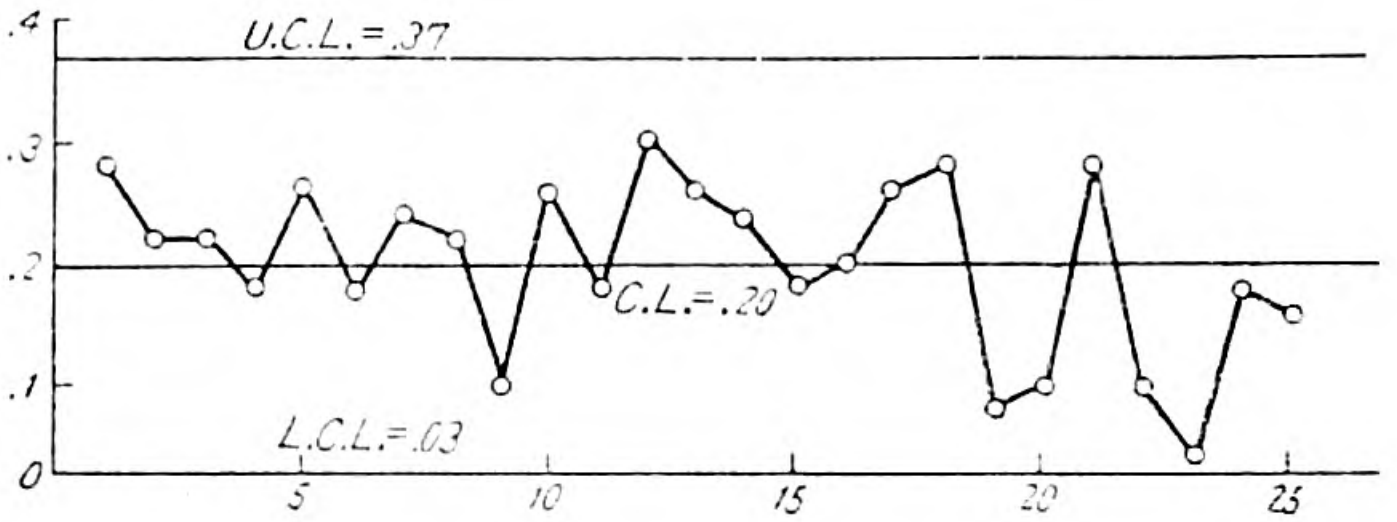


Fig. 10. p-chart of beads.

Now plot the 25 points, then you can connect them. By so doing you will find it easier to see the trend. Here you find no evidence of lack of control. You will find that no point falls out of the control limits. So you will see that although there does exist quite a lot of variability none of it is out of control — we have as much uniformity as is possible under the conditions.

Repetition of this process under excellent conditions will still give you variability. But without the control limits you cannot say whether there is too much or too little variability.

It is difficult to attain statistical control even in a laboratory experiment like this one, so you should have high respect for a manufacturing process that has been brought to a state of statistical control.

Suppose that the results of a factory inspection on a certain day shows up like this. (points to a point near the upper limit) You feel you have a high percentage of detectives, so you change the raw material or make an adjustment in the machines. This is a change in the process. Now, suppose we do this; then on the next day we get a low fraction of defectives.

What would you think?

Sometimes the superintendent of a factory will find that he is producing a high percentage of defective articles. He makes an adjustment. Then, if the percentage is low the next day, he is very proud of himself. He thinks that the changes he made were effective. But remember we are talking about points falling within the control limits. The low point might have occurred even if no change had been made. What mistake did this superintendent make? I want someone to tell me.

Student: This is mistake (a) — Hunting for trouble and making adjustments when no trouble exists.

Deming: Very good, excellent. The chart shows that no trouble existed in the first place.

Now I want to tell you two very important ideas. Please note that the control limits for p depend only on p. In the case of $\bar{X}$ the control limits are set by the range or spread. In the case of p the limits come only from p.

To make sure of the next ideas, I am going to draw the central line on this chart.

I am going to ask you a question. This is the second idea, but it will be in the form of a question. What is the approximate value of this central line p, or average fraction defective?

Student: 20 per cent.

Deming: Good. This is an extremely important percentage.

Now the central line of 20% is sometimes, or in some books, called the *Process Average*, abbreviated PA. Now I have a very important question. Suppose I want less variability. What can I do? I want you to answer that question. I want an answer regardless of whether it is right or wrong.

Student: Increase the size of the sample.

Deming: That is very good. Splendid. However, suppose I wish to keep the sample size the same — 50?

Student: Then it would be impossible to get less variability.

Deming: That is right. We can not get less variability. This kind of variability represents the limit of uniformity. Now I think anyone who can understand this principle has already learned a great deal about quality control.

Now, what is a control chart? It is a tool. A tool to distinguish between controlled and uncontrolled variability. Without control limits you have no tool for deciding whether you have too much or too little variability.

I will now ask you to tell me one more thing. What can I do to achieve points like in this picture?

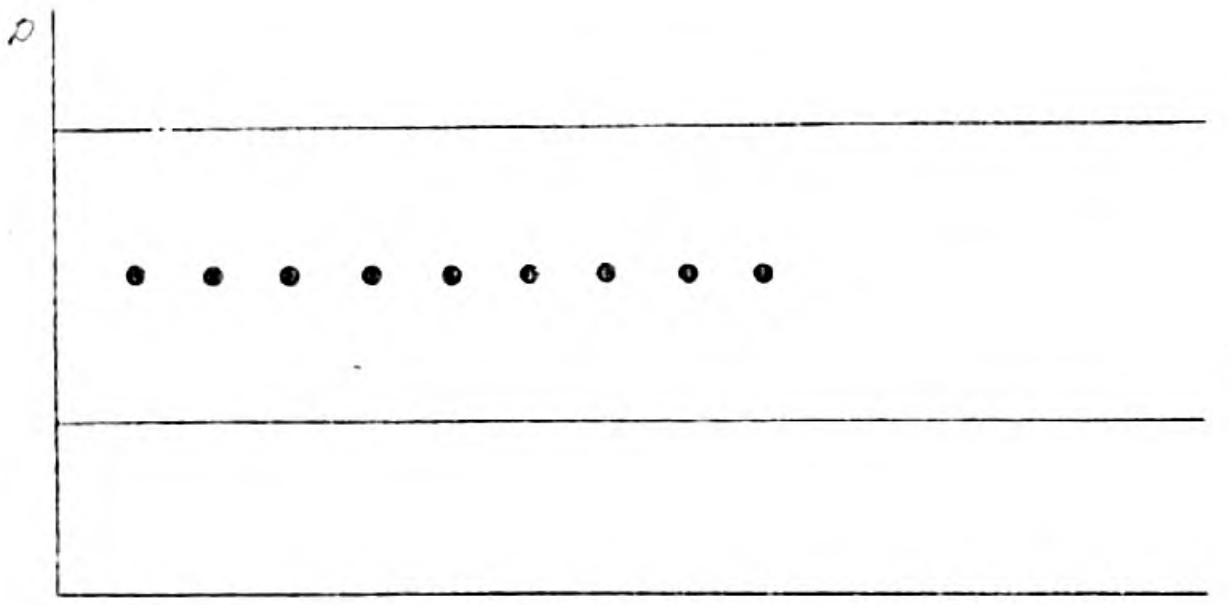


Fig. 11

They all come to the same level of 10 per cent. Extreme uniformity at 5 defectives out of 50. I will show you how this can be done. I just pick out 5 red beads and 45 white ones. But what does this represent? Is it an honest production process? Yes, but it requires accurate inspection of every item, and is usually too expensive.

LECTURE 5

CONTROL CHART METHOD OF CONTROLLING QUALITY DURING PRODUCTION

Now we are going on to the use of the control chart for controlling quality while a product is actually being made.

Diagram—Fig. 1, p. 5. Z1.3, showing two purposes of inspecting a sample from a current lot of product.

The diagram I have drawn on the blackboard shows the two purposes of inspection during production. The two purposes represent two problems confronting production engineers.

Purpose A: *Action on the lot.* What shall we do with the lot thus actually produced? What can I do? We may inspect further. We may sort the good from the bad; we may reject or take all. Now this has to do with a subject of study called *Acceptance Sampling* to which we shall come later in this course.

Purpose B: *Action on the process.* As a result of the information gathered from inspecting the product we may either leave the process alone or make adjustments, retain the operator, change raw materials, etc..

Since we have covered a great deal of what is stated in these pages (p. 5 p. 6 p. 7, Zl. 3) we shall skip some paragraphs and go on to *Section* 5. *Need for keeping records of quality.* I have already mentioned that you must have records of all the testing and inspecting you have done, for without data it is impossible to plot a control chart. Where records have not been kept in the past, one of the first things to do is to start keeping records of the inspection results showing production sequence, and in a form that will permit grouping the data, and examining them by the control chart method of analysis.

Section 6. gives the different types of quality characteristics. Samples may be single articles or test specimens. For these three different kinds of records we may plot three different kinds of control charts. If you have actual measurements you may plot a chart for $\bar{X}$ or R, as we did. If you have a record of the number of defectives as compared to the number of articles actually inspected, you plot the chart for fraction defective p. If you have a record for the number of defectives in each sample you may plot the chart for c.

If you have the actual figures the X and R chart is usually preferable.

Now I shall give you a few minutes to study Fig. 2 and 3 (p. 14 and p. 15, Zl. 3).

Now please turn again to your book. The chart on page 33 (Fig. 8.) is a control chart for p. In this case, the higher the fraction defective the lower down it is on the chart. Some people like it that way. I don't care which way it is done. Now if you will look at the chart you will find that the sample sizes vary from time to time. In our experiment today the sample size was kept constant at 50. But in actual practice the sample size must sometimes be small—say for instance when the output is small. In that case you may compute separate control limits for whatever sample size you have.

In the chart, the dark lines show the control limits. You see what happens when you have a small value for n (note limits for July 27.....). Your fraction is big, so if you have a small value for n your quality limits become larger. Here n was limited by the output which was so small in comparison to the average sample size that special control limits had to be computed to prevent false indication of lack of control.

I will tell you what happened. The records of the number of defectives come from the inspection department. Well, there

was a holiday on which neither work nor inspection was done. But instead of leaving that day out of the chart, they filled it in by using the amount of product inspected on the day after the holiday.

The ground that we have covered today is very important. I want you to review it well, and I want someone to bring me inspection data showing fraction defectives.

Be sure that you understand that quality control is not merely the plotting of charts. This chart was plotted, then *used* to discover what the cause of variability was; then attempts were made to remove that cause. The engineer was partially successful. He is on the way to quality control. No improvements are made if you don't try to remove the cause of uncontrolled variability. Sometimes the action you need to take in order to remove the cause of variability is extremely difficult and complicated.

In several factories in America control charts are placed on the wall of the factory where all the workers may study them. After a few weeks the workers begin to understand the charts without explanation. They find the causes of trouble, take corrective measures and improve the process. That is a good method when any possible carelessness on the part of the workers is a cause of variability.

But as I said, in many cases it is much more complicated. The cause of variability itself may not be complicated. It may not be any carelessness on the part of the workers. In that case it doesn't do any good to put the charts on the wall of the plant. For the workers have no control over the cause.

Here we have alive, up-to-date examples of control charts and their use. You will notice that in May and June the charts really directed the engineer towards the cause of variability.

Now, I want to talk about the *establishment of control limits*

for future use (as a guide to action).

When you have finally attained statistical control over a process or have come very close to such control, then you will be able to set control limits on the basis of recent data, which you will project into the future.

Look at Fig. 8, on page 33 (Zl. 3). You will see certain points that fall outside the control limits. As soon as you find such points you will look for trouble—the cause of variability. When you have found the cause, remove it. Then you will have less variability. Now, it often happens that after removing one cause, the trouble is not corrected entirely. You may have several causes of variability combining to throw a point out of control.

The standard level, in the case of this particular chart was set at 2.65, the process average worked out from data available in the months prior to June 1, when the chart was started. In other words 2.65 is the *adopted value* p' for future use; and it was used as the standard level through June, July and August.

With this chart the control limits were first set and projected into the future. When you do this you can use the same limits for as long as you wish or change them if that is desirable. Now the change that was made early in July is somewhat confusing: the process average remains the same at 2.65, but the smaller value for n, because of smaller average daily output, necessitated a change in the limits

On the first of September it was decided to use a new process average. This new standard level was computed from the actual process average for June, July, and August. Then from September 1 this new level was projected into the future. The new level p' was 2.4. The expected average daily output for September was 1,200. That was what the superintendent

thought on the first of September. Actually it turned out to be 1,208 per day, so it was very close.

Now, I'm talking about control of quality during production. When we plot a chart, we put in the points from day to day or from lot to lot, as quickly as possible. That is because finding the causes of variability is like trying to catch a thief. The resemblance is pretty close, because the cause of variability is indeed robbing you. The best way to catch a thief, is while he is in action—not a month or more later. So for controlling quality during production we must set up standards and control limits. These we project into the future, then the points are plotted day by day, as soon as possible, next morning if possible, or as soon as the inspection record is completed. Then when a point goes out of control, you need look back only a day or so. History is still fresh in your mind. You remember what happened. If you wait too long, nobody remembers what conditions existed on that particular day, or how it was possibly different from any other day. The most insignificant of changes in the process or in the workers might be a most significant adjustment of some sort that will produce noticeable changes in quality. If you wait too long you may be too late.

Now, I wish that you would look at Fig. 6 on page 24 of Z1. 3. You see points No. 31, 34, and 40 outside the control limits. The foreman started an investigation immediately after point No. 31 was plotted and found it to be out of control. By the time point No. 45 was plotted he had discovered the cause—faulty tension in a spring; he replaced it by a weight for more unifrom tension, and thereafter the uniformity appeared to be much improved.

Now there are two ideas I want you to be sure to understand. They are connected with each other, yet I hope you will distinguish between them.

If you have 25 points plotted on a chart, based on data from the past, you may analyze these points, that is, draw the control limits and decide whether you had established statistical control in the past. If you find points out of control, then you are sure that there are troubles, and you try to look back to discover the causes and remove them.

The other idea is the use of the control chart during production. Adopt the control chart on the basis of what the process can do. To use the chart during production you must plot the points without delay. As I said before, if you want to get at the trouble it must be soon, like catching a thief in the act. .

Question: To plot a control chart from data available from the preceding two or three months, is it necessary to use all the data including those pertaining to causes of variability? May we eliminate the points that are out of control?

Deming: It would be all right to throw away those points if you have been able to discover the cause. Never discard data simply because they come too high or too low.

Question: How could we control, that is plot a control chart for bulk material, some chemical for instance, where each lot is of considerable size, and when each lot takes some time to process?

Deming: I think that can be done. In fact I am positive there must be some way of placing the process under control. Now, in some cases I see that we may not be able to get enough samples within a reasonably short period of time. But there must be some place in the process where an inspection point can be set up and samples can be drawn. I will explain.

We have a production process producing some bulk material, some chemical for instance, and suppose it flows in a pipe. You may if you wish draw samples from an outlet on the pipe.

This can be done in some cases, but I do not claim it will fit every process.

Now this is done:

Draw off samples of the product four times daily, perhaps at 10:00, 12:00, 14:00, and 17:00 hours. I would vary the time from day to day. It need not necessarily be a percentage; you may use any scale you wish. Test these samples every day. So every day you get four numbers or percentages. Take these test results as individual values, X_1 X_2 X_3 X_4, to obtain a sample or subgroup with $n=4$. This would permit the plotting of one point each day on the $\bar{X}$ and R chart.

Now this is just one suggestion. It might not work in some cases, but actually it will take care of 95% of the problems. The control limits are set by the sample size; and there is no reason why you cannot obtain enough data for two points a day.

Question: In many mechanical factories it is possible to have complete control on the output. But in the case of chemicals where yield from a certain amount of raw material is a crucial factor it seems difficult to exercise control.

Deming: Suppose you take four successive yields and get four numbers, X_1 X_2 X_3 X_4, and treat them as a sample. You might prefer to use only three successive yields to form a sample so that you can get started quicker. You might even use a sample of two.

Questioner: With the case I have in mind the process takes at least ten days to obtain a yield. Therefore plotting a chart will take too long.

Deming: Well I realize that in that case it will take forty days to get one point plotted; so it will take many many days before 25 points can be plotted on the chart. But there must be some way of controlling the process, and if you have records of

the control you will be able to make a control chart. Many operations in the manufacture of the product warrant study with the use of control charts. Now I do not know the process, but if there is any uncontrolled variability it will probably be found in the quickest way be utilizing the control chart. I believe that if you will think about the matter, keep my suggestion in mind, and try to find out how all the important operations are being performed by using $\bar{X}$ and R you will surely discover something.

If you are using percentages, as was suggested in the previous question, just use the four numbers that you get. Someone has asked if it would be permissible to use just one test result as a sample. That is impossible, because then you get no range. In certain cases you may have to be satisfied with only two individual values per sample, but never one. However, single tests may be plotted as a *run chart*, without control limits, and a run chart is extremely useful.

Now, I wish to take a little time with you to collect data for $\bar{X}$ and R. But before we go on, I wish to stress again that quality control is a tool, and that it must be used like a tool, as you would a screwdriver, wrench or any other tool.

Here I have a very simple operation. I take chips from a bowl without looking into it. I think it is a controlled process. We have three bowls containing chips. Now the chips are numbered; and we shall pass the bowls around for each man to pick out a chip without looking. He will then read out the number which will be noted down as data for a sample. Data from five chips will make a sample. Now this is what we are going to do:

Draw the samples. Compute $\bar{X}$ and R. Compute the upper and lower control limits for $\bar{X}$ and R (Table 3). Plot charts for $\bar{X}$ and R. Draw the central lines and control

Table 3
(Universe A)

For $\bar{X}$ and R–chart

Name of Co.

Date 7. 13

Article Chips

Lot number 1

Operation

Name

Type of measurement

Remarks Universe A

Sample number						$\bar{X}$	R	σ
1	-2	-3	-2	-2	-2	-2.2	1	
2	-4	2	2	2	2	0.8	6	
3	0	1	2	0	0	0.6	2	
4	2	0	1	-1	1	0.6	3	
5	-1	-1	-3	3	2	0.	6	
6	-3	3	-1	0	-1	-0.4	6	
7	2	0	0	-1	-2	-0.2	4	
8	-1	2	3	2	2	1.6	4	
9	2	-2	2	-1	-2	-0.2	4	
10	2	-1	-1	4	-2	0.4	6	
11	-1	2	2	-1	1	0.6	3	
12	0	0	2	1	2	1.0	2	
13	2	1	0	0	-1	0.4	3	
14	-2	0	1	1	-2	-0.4	3	
15	2	-2	-2	1	0	-0.2	4	
16	-1	-2	-2	2	1	-0.4	4	
17	1	0	1	-2	-3	-0.6	4	
18	2	0	2	2	-1	1.0	3	
19	2	2	-1	-1	-2	0	4	
20	-1	1	2	0	-1	0.2	3	
21	-4	1	0	0	1	-0.4	5	
22	-1	-2	1	0	1	-0.2	3	
23	2	-2	0	-2	3	0.2	5	
24	2	-2	-2	3	1	0.4	5	
25	3	2	-1	-2	-2	0	5	
Total								
Average						$\bar{X}$=0.104	R=3.92	$\bar{\sigma}$=

On the $\bar{X}$–chart: Central line, $\bar{X}$= 0.104 A_2R=

UCL, $\bar{X}+A_2R$= 0.104 + 2263 = 2.37

LCL, $\bar{X}-A_2R$= 0.104 - 2263 = -2.16

On the R–chart: Central line. $\bar{R}$= 3.92

UCL, $D_4\bar{R}$= 2.114 × 3.92 = 8.29

LCL, $D_3\bar{R}$= 0

Table 4
(Universe B)

For $\bar{X}$ and R-chart

Name of Co.

Date 7. 12

Article Chip

Lot number 2

Operation

Name

Type of measurement

Remarks Universe A

Sample number						$\bar{X}$	R	σ
1	-1	0	1	1	-1	0	2	
2	-2	1	-4	1	-2	-1.2	8	
3	1	4	-2	-2	0	-0.2	6	
4	-2	1	0	3	-1	0.2	5	
5	1	4	1	-2	1	1.0	6	
6	1	1	-1	-4	-1	0.8	5	
7	-1	-1	2	0	-2	-0.4	4	
8	1	-2	-1	-2	-1	-1.0	3	
9	1	-1	-1	-1	0	-0.4	2	
10	-2	-1	2	-2	-1	-0.8	4	
11	1	-2	2	-2	-1	-0.4	4	
12	-2	1	0	-2	1	-0.4	3	
13	1	0	2	1	2	1.2	2	
14	2	-2	-2	0	2	0	4	
15	3	0	-1	-2	-2	-0.4	5	
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
Total								
Average						$\bar{\bar{X}}=$	$\bar{R}=$	$\bar{\sigma}=$

On the $\bar{X}$-chart: Central line, $\bar{\bar{X}}=$ $A_2\bar{R}=$

UCL, $\bar{\bar{X}}+A_2\bar{R}=$

LCL, $\bar{\bar{X}}-A_2\bar{R}=$

On the R-chart: Central line, $\bar{R}=$

UCL, $D_4\bar{R}=$

LCL, $D_3\bar{R}=$

Table 5
(Universe C)

For $\bar{X}$ and R–chart

Name of Co.

Date 7. 13

Article Chips

Lot number 3

Operation

Name

Type of measurement

Remarks Universe B

Sample number						Σ	R	σ
1	3	1	-1	2	0	1.0	4	
2	4	4	3	3	3	3.4	1	
3	4	2	0	2	4	2.4	4	
4	4	1	3	-1	1	1.6	4	
5	1	-1	1	1	2	0.8	3	
6	4	4	3	0	3	2.8	4	
7	4	6	2	6	4	4.6	4	
8	4	4	2	4	3	3.6	3	
9	4	4	0	1	0	2.0	4	
10	3	2	2	4	1	2.4	3	
11	4	2	2	1	2	2.2	3	
12	2	2	1	1	2	1.6	1	
13	0	2	2	-1	6	1.8	7	
14	2	2	2	4	3	2.6	2	
15	1	3	1	1	0	1.2	3	
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
Total								
Average						$\bar{\bar{X}}=$	$\bar{R}=$	$\bar{\sigma}=$

On the $\bar{X}$–chart: Central line, $\bar{\bar{X}}=$ $A_2R=$

UCL, $\bar{\bar{X}}+A_2\bar{R}=$

LCL, $\bar{\bar{X}}-A_2\bar{R}=$

On the R–chart: Central line. $\bar{R}=$

UCL, $D_4\bar{R}=$

LCL, $D_3\bar{R}=$

Table 6
(Universe D)

For $\bar{X}$ and R-chart

Name of Co.

Date 7 13

Article Chifo

Lot number 4

Operation

Name

Type of measurement

Remarks Universe C

Sample number						$\bar{X}$	R	σ
1	-4	-1	3	-6	1	-1.4	9	
2	0	2	7	-1	2	2.0	8	
3	1	-7	-7	0	1	-2.6	8	
4	2	0	-2	2	-4	-0.4	6	
5	-4	-1	2	2	-3	-0.8	6	
6	-1	5	2	0	-2	-0.2	11	
7	-5	0	3	-4	-3	-1.8	8	
8	-6	5	0	-3	2	-0.4	11	
9	-3	2	5	5	2	2.2	8	
10	1	-2	-2	-1	-1	-1.0	3	
11	3	0	1	-3	7	1.6	10	
12	-2	-3	0	-1	0	0	5	
13	-3	-4	-2	-3	-2	-2.8	2	
14	4	1	-2	1	-2	0.4	6	
15	-4	-4	-3	3	4	-0.2	8	
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
Total								
Average						$\bar{\bar{X}}=$	$\bar{R}=$	$\bar{\sigma}=$

On the $\bar{X}$-chart: Central line, $\bar{\bar{X}}=$ $A_2\bar{R}=$

UCL, $\bar{\bar{X}}+A_2\bar{R}=$

LCL, $\bar{\bar{X}}-A_2\bar{R}=$

On the R-chart: Central line. $\bar{R}=$

UCL, $D_4\bar{R}=$

LCL, $D_3\bar{R}=$

limits. Project the control limits into the future for 45 more points. Draw 15 more samples from Universe (bowl) A, and plot the points one by one as they are drawn. (Table 4)

Same for Universe B (Table 5)

Same for Universe C (Table 6)

Are Universes A, B. and C all alike?

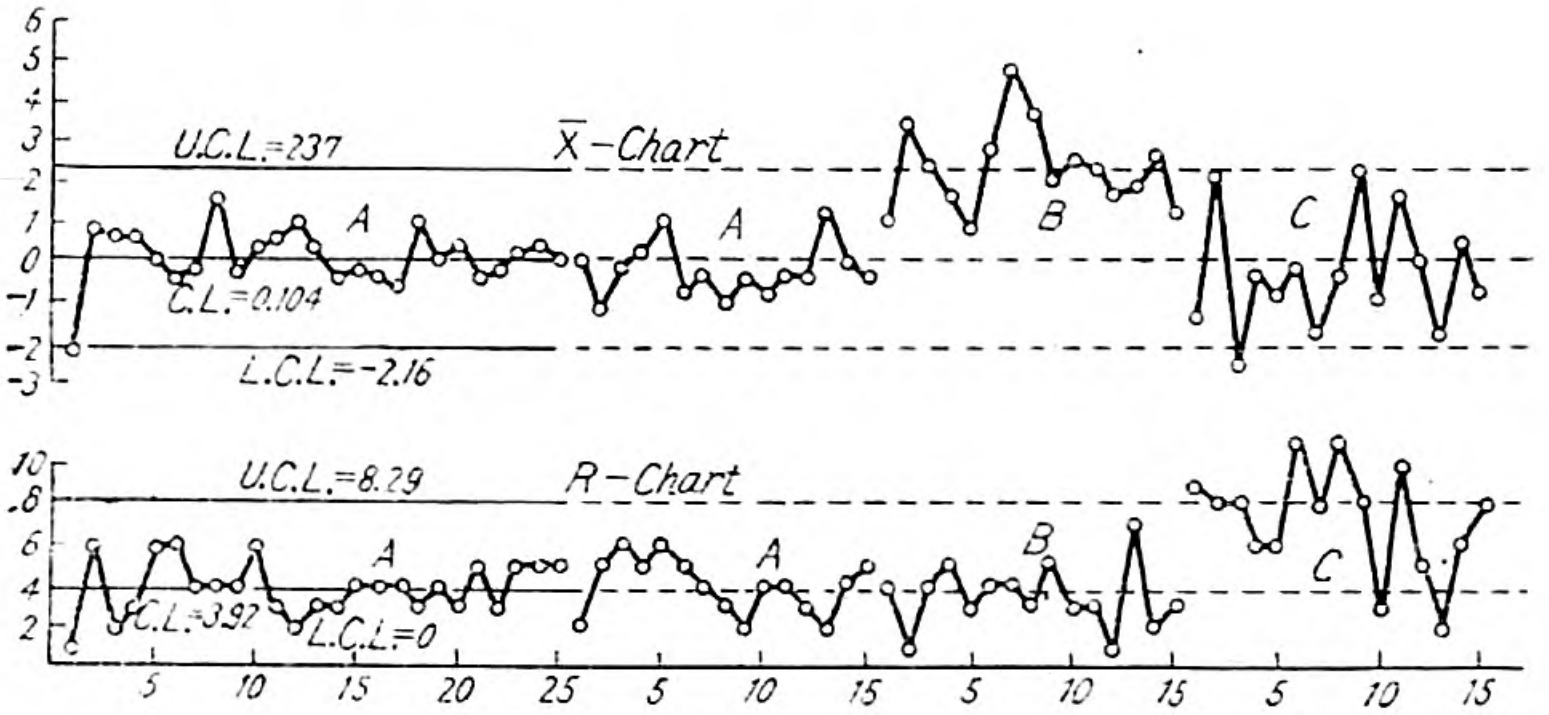


Fig. 12

Here, in Fig. 12, we have the charts plotted from this experiment. They show you how a control chart will detect changes. Universes A, B, and C contained three different sets of chips marked differently. By drawing the chips from B instead of A we saw what the control chart detected. Something happened. The change from A to B is equivalent to a change in the process—different raw materials; new machines; etc. Likewise a change from universe B to universe C is like a change in the production process. So *the control chart will detect a change in the production process if the changes are worth detecting and eliminating.*

Are there any questions? Is it clear to you what our last experiment was about? Now, I wish to ask you a question. Suppose I put all three kinds of chips, A, B, and C, in one bowl, stir, draw 25 samples and compute new control limits. Do you think that such drawings would indicate a state of statistical

control? What do you think?

Student: A state of control would exist.

Deming: I agree. Suppose we try and see.

Now, the universes A, B, and C could be 3 machines, or 3 sources of raw materials, 3 different workers, or 3 different sets of workers. Now if you keep them separated you see that you are able to detect the differences between them. But if we mix up the three together, we shall not be able to detect any difference between A, B, and C.

The aim of this experiment is to show that you should keep your control charts properly segregated: that is, keep your source of raw materials, machines, and workers separated when you work on data obtained from them. Otherwise you will not be able to detect the differences.

Now let us look again at the charts (Fig. 12).

The first 25 points were plotted from samples taken from universe A; and from these points we can judge that the process apparently is in a good state of control. From the 26th point on, we have the chart projected into the future. That is why the central line and the control limits are indicated by broken lines. Now, using these limits, we plotted more points for universe A. We discover that there is no noticeable change, and that the points continue to indicate excellent control. However, when we attempt to use the same control limits for universe B, things do not work out so well. The central line for universe B—the level of $\bar{X}$ for the samples drawn from universe B—apparently should be higher up. But the range, as indicated in the R-chart, seems to come within the extended limits.

Next, we tried the same with universe C. In this case, we discover that the average level of $\bar{X}$ is about that same as for universe A, but the points above and below the central line have a much wider vertical spread than with either universe A or B.

Moreover, if we use the same limits for range as those for universe A, we find the points for universe C falling out of control. In my judgment, universe B is very similar to universe A, with the spread within samples about the same. Only $\bar{X}$, the central. line, is at a higher level. With universe C, although the central line is at about the same level as for universe A, there is much more variability within samples.

From these charts, I believe we can learn something about the differences in make-up of universes A, B, and C. Later on, I shall give you the actual values; but I think that you now perceive how the control charts can give you a rough idea of the characteristics of a universe or a process even before you know the actual values.

Now here are the values which made up the compositions of universes A. B, and C:

Number written on the Poker Chip	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
Frequency of occurrence of the numbered poker chips A							1	3	7	9	10	9	7	3	1						
B									1	3	7	9	10	9	7	3	1				
C	1	0	1	1	2	4	6	8	10	11	12	11	10	8	6	4	2	1	1	0	1

	Mean	Standard Deviation
Universe (bowl) A	0	1.79
Universe (bowl) B	2	1.79
Universe (bowl) C	0	3.48

When our universes (processes) are separate, like we had in the experiment, we can get the charts to help us detect the differences existing between the universes or processes.

I want you to see how important it is to keep the possible sources of variability clearly separated by themselves. Now we also observed that if we mix the three processes we shall no longer be able to detect differences among them. But in thus making one process out of the three we may still have good

control.

If we mix the chips that represent the three processes. and perform a new experiment, we shall find that the mixture shows statistical control.

Now, are there any questions?

Question: The first point on the chart (Fig. 12, A) is out of control. Please explain.

Deming: I think that is a very good observation. My explanation is that someone made a mistake. I think you should make a note on your papers that the point is out of control. Now you may have a different explanation, but the process was a controlled process so the point out of control could only be the result of a mistake. Look at it from another point of view. Figures come not directly from the production process but through the inspectors who can make mistakes or give wrong values. I think we have a very important lesson here. When any variability is detected, the first place to look for the cause would be in the inspection and records procedure. Remember always that the figures that you are working with are the result of a great number of events. One of them is the production process; another is the inspection and recording. And the data have gone through a lot of things before you plot a chart.

I want to tell you a story about what actually happened in a manufacturing plant that was producing men's coats. The company was running three shifts a day. There was the day shift, and the night shift. Now, in the section we are talking about, each shift had 20 workers, 1 foreman, and 4 inspectors. Now, it was observed that the night shift made fewer mistakes, had fewer rejects. The day shift had more rejects than the other two; in other words the day shift was the worst and the night shift was the best. I am sorry I don't have the records here with me, but from what I remember the day shift had 30% more rejects than

the night shift. The problem became very complicated because the workers changed shifts every two weeks. But it was noted from the records kept over a period of more than six months that there were consistently more rejects from the day shift regardless of the change in workers every two weeks. The inspectors also noted that over the six month period there was no significant difference between the number of defects produced by the three different shifts. However, finally the explanation became clear. Can you guess what it was?

Student: The inspection itself at night time was not so good as it was during the day.

Deming: It is a very good answer. The lighting was not so good as in the day time. The day time inspectors could easily find the defects. Now I am trying to show the importance of always remembering that the figures you are getting are the result of a chain of events.

Now to get back to the chart and to the point that is out of control. I know we had the same bowl: there was no change in the bowl. Neither was there any change in the chips; and since the stirring was done pretty carefully my explanation would be simply that there was a mistake in reading the figure. A further explanation would be that it was the first sample, and that we had not yet quite learned how to read and record the figures. I am very glad that this happened because you see right in front of you what can happen if you do not arrange things very carefully.

Perhaps you will now agree with me that the achievement of statistical control in a real manufacturing process calls for real effort, and is a real achievement.

There was a book published by Dr. Shewhart in 1938 (W. A. Shewhart: *Statistical Method from the Viewpoint of Quality Control*). It showed data from a production process. The data

which came from 204 fuses indicated uncontrolled variability—a production process without control. But when he wrote those 204 numbers on chips, put them in a bowl just as we did, and plotted 25 points out of samples of 4, the process showed beautiful control. The lesson here is that the data should always be taken in the order of manufacture. Retain the order in which the articles are made. Do not mix the articles or the days. A sample should come from the same lot, same day, same machine, same worker or group of workers, same week. They should come from the same source, as close together as possible.

Now there are many problems in connection with the choice of the data that go into a sample. When you have separate articles it is easy to pick out the defects or the defectives. But when you are working on long rolls of paper, film, cloth or sheet metal it is different.

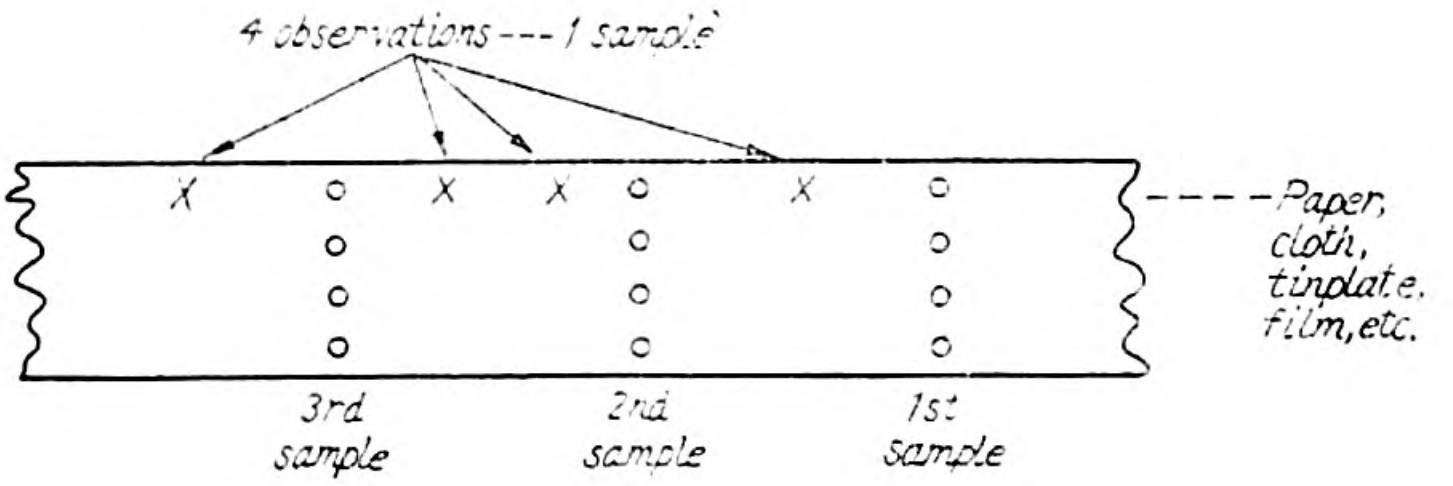


Fig. 13

You can take four observations lengthwise and call that a sample or you can take four observations crosswise and call that one sample. Which way is correct depends on what you are trying to find out. If you are trying to discover variability along the elongation then you make your 4 observations crosswise; if you are looking for trouble across the sheet then you make your observations lengthwise.

The aim of subgrouping or making samples should be *to get the least amount of variability within the sample itself* and *to get*

the greatest possible amount of variability between the samples.

I know that some of you have studied the theory of sampling. You may have heard or read about stratification and subgrouping. It is the same thing. "Stratification" comes from the latin word *stratum* which simply means layer. Now, the production engineer must suspect first of all that there are differences between machines, between lots of raw material, between workers, between one half of an hour and the other half. One of the first problems is to decide where variability may come from—in other words, decide the layer. Think of those layers. It might be one half hour or another half hour, or it might be a certain machine or another machine. First of all you must decide what causes uncontrolled variability — determine which layer, then decide on the samples to be taken from that layer. Make some observations in the layer that you have chosen, call the result sample 1; then go on to the next layer.

The samples of 4 or 6 successive articles every half hour should tell you whether the conditions remain the same this day or whether the cause of uncontrolled variability is present occasionally at another time. Every point should be carefully noted as to which came from a certain half hour, or a certain machine, or a certain lot of raw materials, or a certain group of workers. Now again I say: *Do not mix the sources when you plot your points.* If you take a sample for a certain half hour, the next sample for another half hour should come from the same machines, the same lot of raw material, the same group of workers. Do not vary the machines or the other sources. Now as you go on plotting your chart you may think that there are causes of variability that come and go and come again. In time, you may be able to detect the cause; that brings us to another important thing—repetition.

The problem of subgrouping or stratification or choosing a

layer is a major *engineering judgment* on where the cause of variability lies. When you decide just what could possibly be a cause of trouble, what is the cause of more variability than is desirable, then the layer, the stratum or subgroup becomes obvious to you.

When you see a point out of control you must ask yourself this question: What was the difference here?

Now you may not know. You may not be able to answer the question. But *with a point out of control you do know that there was a difference.* Take the case of our experiment. We found the first point out of control. There was a difference. But what was it? Was it the bowl? No. It was the same bowl. The difference about this first point was that it was the first sample. And the first 5 people that performed the experiment were not as accurate in calling out the numbers that they read as the others were.

Student: In connection with the first point which is out of control (Fig. 12, A), is it not exceptional? Would such a thing happen more than once in a thousand or ten thousand tries?

Deming: I would adopt the rule to look for a source of uncontrolled variability every time a point goes out of control. I will be right 999 times out of a thousand, or better. Thus I have a pretty good rule. I don't know a better rule. *The control chart is used as a tool — a tool for action.* Look for uncontrolled variability, or don't look for uncontrolled variability. The question is not really whether I could possibly be wrong one time in a thousand. I believe that it is better to look at it this way: *the control chart is a tool for obtaining the most economical manufacturing method.* Look for the trouble and its explanation, and try to remove the cause every time a point goes out of control. But leave things alone for the point that is inside. You will thus achieve the greatest economy in manufacturing.

In that way you will minimize the net loss from the two kinds of mistakes (a) and (b) that we talked about at the beginning of this course (page 13).

Now, keeping in mind that the net loss must be made as small as possible, the separation of the control limits on the control chart comes to just about 3 *sigma*. What we must adopt is a definite ***rule***: draw a limit for taking action or not taking action, and stick to this rule day after day. It is a good rule. I am not sure that I understand exactly how the questioner feels about it, but the control chart is a tool for obtaining the minimum net loss from the two possible mistakes. But what I have been trying to explain is that it took many years to build the idea. Now I hope it won't take you as long as it did me to understand how to use the control chart as a tool. I believe that if I could teach only one thing about quality control, that would be the understanding of the control chart as a tool. This is an essential concept. Are there any other questions?

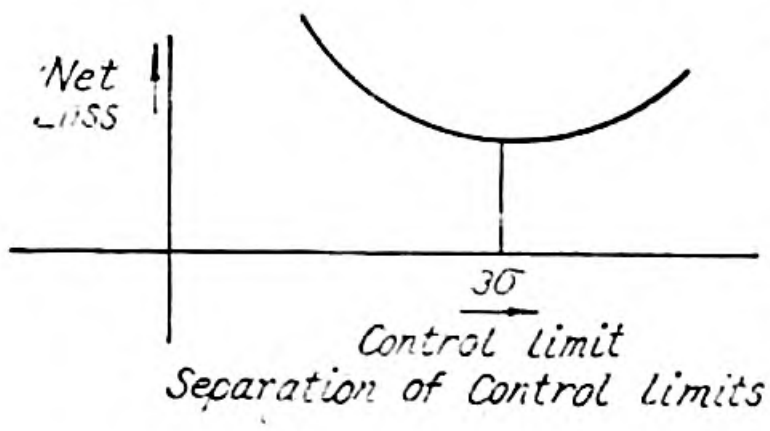


Fig. 14

Student: In the $\bar{X}$- chart for bowl A, I find several points in succession are below the average. Does that mean anything significant?

Deming: I think several points in succession below the average can be taken to indicate something, perhaps uncontrolled variability. I have a suspicion that the chips were not stirred properly. This illustrates, I believe, how careful you must be in spite of how simple an experiment or process you have.

Student: According to our textbook, American War Standard Z1.3, the 3 sigma separation for control limits is cited as giving the minimum net loss. Should this not vary with the

type of industry, and also with the level of technology?

Deming: There is such a possibility. But unless there is a very special reason, the separation should not be changed. Once a rule is established, it is better to stick to that rule. I myself have tried different values, but I know now that it is better not to. In certain cases, it happens that exceptions must be made. The rule, however, was established as a result of years of experience and it has been demonstrated that it is a serviceable and useful rule. The less we tamper with it the better.

Now I believe I should make a remark at this point. I wish I had made it yesterday. It is in connection with the p-chart. It is simply this: For the $\bar{X}$ and R chart I told you to use a small sample of say 4 or 5. In other words 4 or 5 individual values will make the samples for the $\bar{X}$ and R chart. But with the p chart the sample size must be much larger. A sample for the p chart should be large enough to give you at least 6 to 10 defectives. It should be at least 5 and preferably 10. If the process average is as low as 1% then the sample should be made up of at least 500 individual values.

Process Average $(\bar{p}.\ a.) = 1\%$

then, if $n\bar{p} = 5$

$$n = \frac{5}{\bar{p}} = \frac{5}{0.01} = 500$$

With such a large sample you have some of the advantages that you have in $\bar{X}$ and R. When only 50 or so parts are produced in a day, it is easy to pick out 4 samples for plotting X and R charts. But for plotting a p-chart, even a whole day's production may not be enough. You cannot isolate out the variability. Therefore the $\bar{X}$ and R chart is often more effective in detecting uncontrolled varability. However the p-chart is extremely important and valuable as an exhibit of quality. And

moreover it is often the first step and a good one.

Now we shall perform another experiment and plot charts. We will collect data and make two $\bar{X}$ and R charts. This time we shall use all three bowls, and we shall never know in advance from which bowl the sample will be drawn. I have here before me three chips marked "A" "B" and "C". I shall draw one of these chips at random for each sample, and you will draw from the bowl I indicate. We will thus mix up the order, although we keep the bowls separated.

〔The experiment is carried out: samples are drawn, and the numbers are noted down.〕

Now compute the control limits for $\bar{X}$ and R. The next step is to plot the chart and see if there is any uncontrolled variability (Table 7, and Fig. 15).

We have before us the chart plotted from the data we got from the experiment that we conducted yesterday. Now, when the points are properly identified as "A", "B" and "C" an interesting thing is observed: On the range chart, none of the points goes beyond the limits, although there are considerable ups and downs; the points for "B" on the $\bar{X}$ chart are nearly all above the central line, though none of the points goes beyond the upper control limit.

You might observe here that the points for "C" on the R chart are very close to the upper limit. Now, any extraordinary pattern or trend may call for further investigation.

Now, if we did not identify the point as coming from "A" "B" and "C" we should not notice anything unusual; moreover no point goes beyond the control limits. Yet we are able to discover something extraordinary about the three bowls. So the next step would be to plot control charts for each of the bowls or universes. But this we have already done. We performed this later experiment to show that even if you mix the three sources on the

same chart, it is wise to maintain the identities and look out for peculiarities.

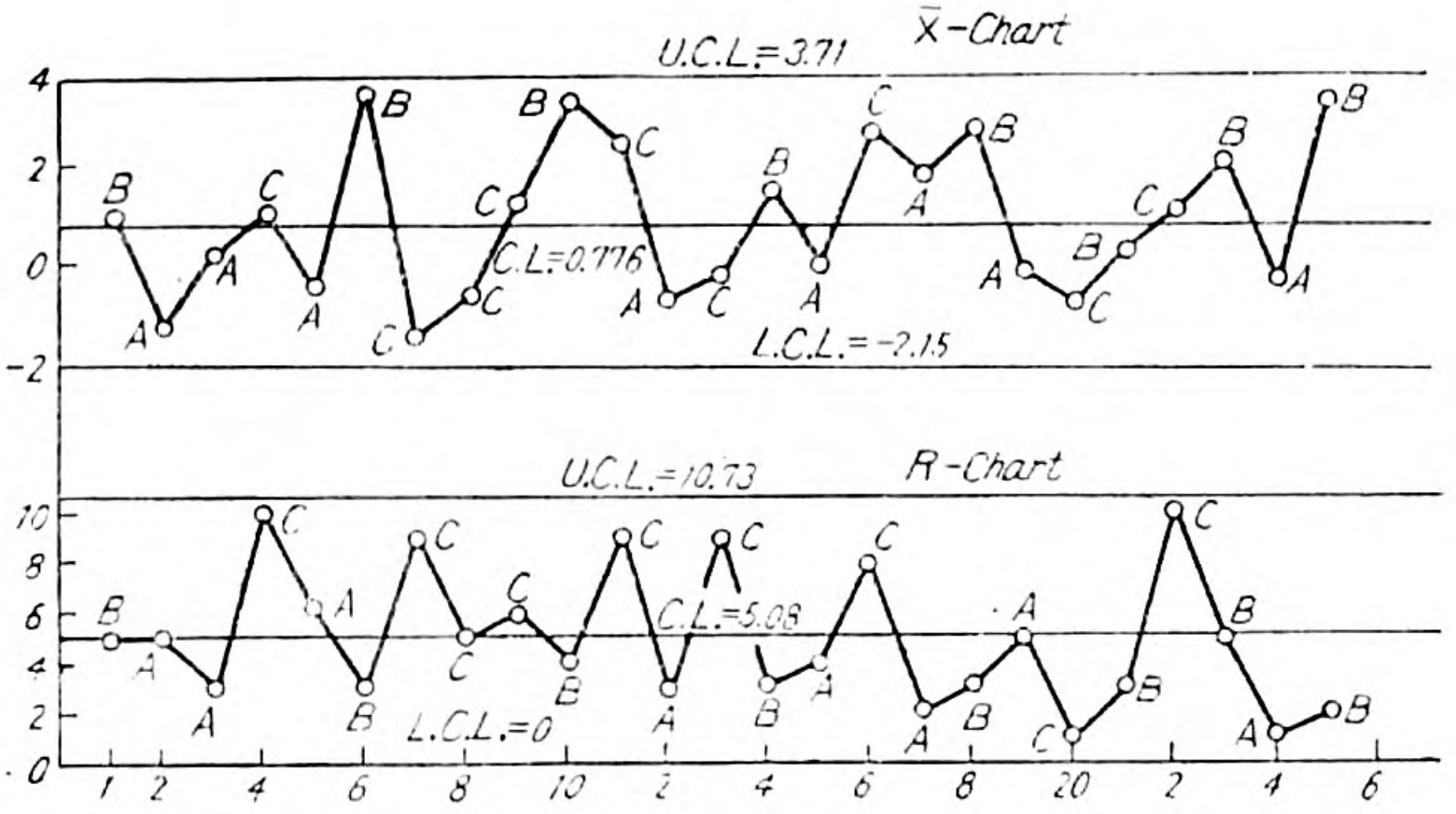


Fig. 15. Universes A, B, C, selected at random

Next I shall draw 25 samples from a bowl containing all of the chips. We shall mix the chips from bowls "A" "B" and "C", put them in one bowl, then draw the samples. Up to now we have experimented with three separate sources of variability —three distinct production processes. But the next experiment is different. Three different sets of chips will be mixed into one bowl and we shall draw samples. In this case I think it will be found that there is no hope of discovering any difference between "A" "B" and "C". Oftentimes in a factory there will be three or more machines producing the same things. Now if the points from each machine are kept separate or identifiable, even if plotted on a single chart, then it will be possible to see what is happening with each machine. However if the points get mixed, and you are not able to tell which come from what machines, then you will be unable to find anything really important.

There are statistical techniques for discovering whether im-

Table 7

For $\bar{X}$ and R-chart

Name of Co.

Date 7. 14

Article Chips

Lot number

Operation

Name

Type of measurement

Remarks Universes A. B. C.

Sample number						Σ	R	σ
1 B	0	4	1	1	-1	1.0	5	
2 A	-2	-4	-1	1	0	-1.2	5	
3 A	0	2	-1	0	0	0.2	3	
4 C	4	-4	1	2	2	1.0	10	
5 A	-1	1	-2	-3	3	-0.4	6	
6 B	2	0	5	5	5	3.4	3	
7 C	-4	4	1	-4	-2	-1.4	9	
8 C	-1	-3	0	-1	2	-0.4	5	
9 C	3	-1	-2	4	2	1.2	6	
10 B	0	2	3	1	4	2.2	4	
11 C	-3	4	6	3	-3	1.4	9	
12 A	-1	-2	1	-1	-1	-0.8	3	
13 C	1	1	-2	-4	4	-0.2	9	
14 B	0	1	2	3	1	1.4	3	
15 A	1	-2	2	-1	0	0	4	
16 C	-2	6	-1	6	4	2.6	8	
17 A	3	1	3	1	1	1.8	2	
18 B	1	3	4	1	4	2.6	3	
19 A	0	2	0	-3	2	-0.2	5	
20 C	0	0	-2	-1	-1	-0.8	1	
21 B	0	1	2	-1	-1	0.2	3	
22 C	-4	3	0	2	5	1.0	10	
23 B	2	3	2	-1	4	2.0	5	
24 A	-1	-1	0	0	0	-0.4	1	
25 B	3	4	2	3	4	3.2	2	
Total								
Average						$\bar{X}=0.776$	$\bar{R}=5.08$	$\bar{\sigma}=$

On the $\bar{X}$–chart: Central line, $\bar{\bar{X}}=0.776$ $A_2\bar{R}=$

UCL, $\bar{\bar{X}}+A_2\bar{R}=0.776+.577\times5.08=3.71$

LCL, $\bar{\bar{X}}-A_2\bar{R}=2.776=-2.14$

On the R–chart: Central line. $\bar{R}=5.08$

UCL, $D_4\bar{R}=10.73$

LCL, $D_3\bar{R}=0$

portant differences exist between the machines. The control chart technique is but one of them. It is well to remember that there are other techniques and that each has its advantages.

Now, the important thing to remember is to *keep the data separated*: nothing can be done if you mix them up together. Are there any questions or any remarks?

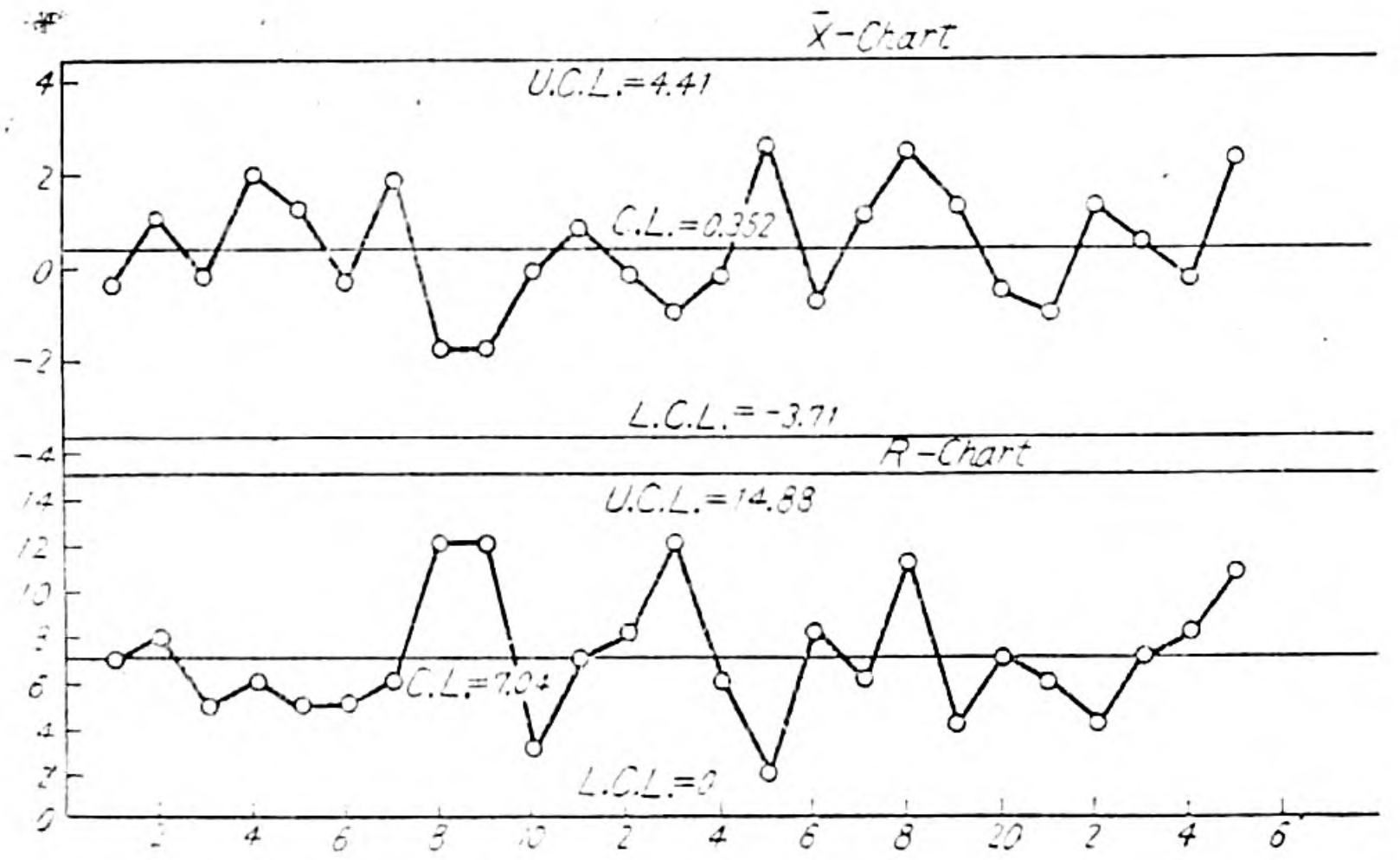


Fig. 16 Universes A, B, and C, mixed.

Prepare your papers for $\bar{X}$ and R. Then plot the chart for the mixture where identification was lost.

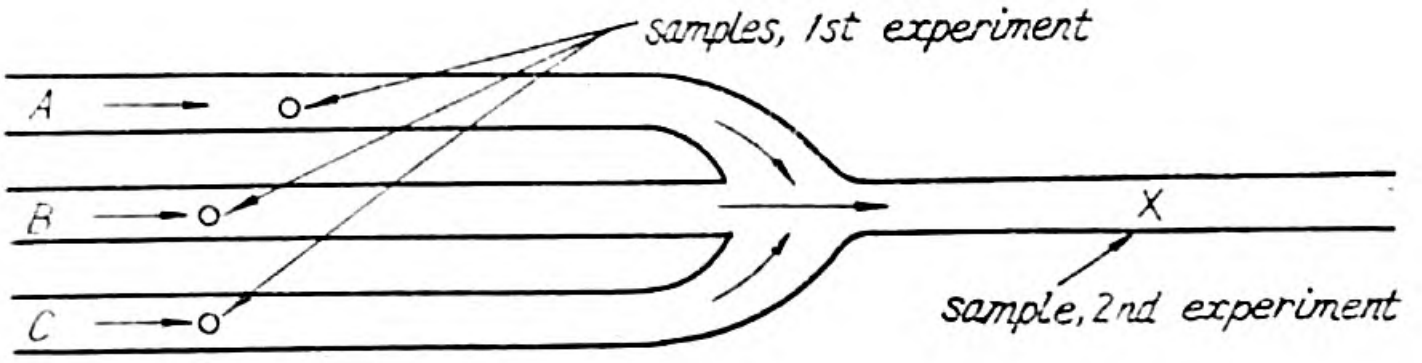


Fig. 17

I have been looking around at the charts you are preparing and I think they are excellent. However, some of them seem to

Table 8

For $\bar{X}$ and R–chart

Name of Co.

Date 7. 15

Article Clips

Lot number

Operation

Name

Type of measurement

Remarks Universes A. B. C. mixed

Sample number						$\bar{X}$	R	σ
1	−4	−2	2	3	1	−0.4	7	
2	4	0	0	3	−3	1.0	8	
3	0	2	1	−3	−1	−0.2	5	
4	2	4	2	−2	4	2.0	6	
5	−1	3	4	1	0	1.2	5	
6	−1	−3	2	1	−1	−0.4	5	
7	−1	5	1	0	4	1.8	6	
8	2	0	−10	−2	1	−1.8	12	
9	1	−8	4	8	−3	−1.8	12	
10	−2	1	1	0	−1	−0.2	3	
11	4	−1	−2	10	2	0.8	7	
12	3	−3	1	3	−4	−0.2	8	
13	−3	4	−1	3	−8	−1.0	12	
14	−2	−2	4	−1	0	−0.2	6	
15	3	3	2	4	2	2.6	2	
16	0	−4	4	−3	−1	−0.2	8	
17	1	0	−2	2	4	1.0	6	
18	0	8	−3	3	4	2.4	11	
19	3	1	0	−1	3	1.2	4	
20	−2	3	−3	3	−4	−0.6	7	
21	−1	−4	2	−4	2	−1.0	6	
22	0	1	−1	3	3	1.2	4	
23	0	3	−4	3	0	0.4	7	
24	0	−2	2	4	3	−0.4	8	
25	3	−1	0	−1	10	2.2	11	
Total								
Average						$\bar{\bar{X}}=0.352$	$\bar{R}=7.04$	$\bar{\sigma}=$

On the $\bar{X}$–chart: Central line, $\bar{\bar{X}}=0.352$ $A_2\bar{R}=$

UCL, $\bar{\bar{X}}+A_2\bar{R}=0.352+.577\times 7.04=4.41$

LCL, $\bar{\bar{X}}-A_2\bar{R}=.0.352-.577\times 7.04=-3.71$

On the R–chart: Central line, $\bar{R}=7.04$

UCL, $D_4\bar{R}=2.114\times 7.04=14.88$

LCL, $D_3\bar{R}=0$

be too wide. Remember that the control chart is not something with which to measure and read figures, but something which gives an indication of the conditions of quality.

Now you will observe on the chart (Fig. 16) that there is nothing to indicate any uncontrolled source of variablity. In other words, what we got when we mixed the chips, stirred and drew the samples was controlled variability. We happen to know that this was a mixture from three different sources, but it is still a controlled process. Now I believe that this experiment, like the ones we have already conducted, will tell us some very important principles. One of these is that you must think before you construct your control chart. Before you sample you must think: Where could there be a source of variability? I suggest you write down the following in your note book, and write it big:

THE CONTROL CHART IS NO SUBSTITUTE FOR
THE BRAIN

First of all you must think: Where could there be sources of uncontrolled variability? Then plot the charts so that you discover this variability if it is an uncontrolled one.

You must *THINK*.

Plot the charts so as to discover variability if it exists.

With today's experiment we know that three different processes, or three machines, were the sources of the samples. Now, if there could possibly be any variability between these three machines "A" "B" and "C" we should plot our charts in the way we did before. In that way we can discover the differences between the machines. When we have once learned from the control charts that three machines are different, or three sources of raw material are not the same, then we can take steps to make changes. We can make changes that will bring the machines into coincidence so that they produce the same results.

But if we do not think first, we may never realize that there is uncontrolled variability. Do not simply look at the control chart and conclude that everything is in perfect condition in your plant. Try out different sources of possible variability; construct charts so as to discover that variability. I can only repeat this many times.

Now I have finished this particular question and I should like to start another topic which will be about the control chart for *c*. But before we proceed, are there any questions?

Student: You said that accurate scaling of the control chart is unnecessary. Is it implied that the figures need not be accurate?

Deming: No. What I mean is that the control chart is not a measuring tool for indicating exact quantities. If the trend is indicated that is enough.

Student: Under present conditions Japanese industries are working on very small lots of raw materials. Therefore the control limits are difficult to set, and it becomes extremely difficult to detect the sources of uncontrolled variability.

Deming: All I can say is: if there is so much variability between lots, which you know causes lack of uniformity in quality of the product, then you already know enough. There will be no need to use a control chart to discover that variability. Remember there is no substitute for brains. Now if you wish to achieve uniformity but can only get raw materials in very small quantities from many different sources, you will have to work that problem out for yourself. You may wish to mix them so as to get a constant uniform mixture. Then you might test your process and product to see whether your brain has brought about the uniformity that you desire.

Student: When you have a point out of control that indicates a source of variability, could not that same source or

cause produce much greater variability?

Deming: Your remark leads me to make a statement that may surprise you. It is very important. Suppose you are computing $\bar{X}$ in a case where there is only a lower specification limit, which you wish to exceed. Now suppose a point falls above the control limits. Would you investigate the process to try and discover the cause? I want somebody to tell me; "Yes" or "No".

Answer: Yes.

Deming: Yes, it should be investigated. But why?

Answer: Because if the cause can be found it could be made use of.

Deming: Any other reason?

Answer: It would not be economical to produce anything better than is needed.

Deming: I agree, but I think I shall have to give the answer myself. If there is a cause of variability which, even though at one time appears above the upper limit, you cannot safely go ahead until you know just what it is because that same cause of variability may later result in low quality and a huge number of rejections.

Now I will go on to the control chart for c; c is the number of defects (*cf.* Zl. 3 *Section* 16 page 18).

This chart is used when the inspection brings out the number of defects in a sample, say a hundred yards of woolen cloth, or a square foot of paint, or a certain length of wire. Well, here let us suppose that we are inspecting a hundred yards of cloth every day and counting the defects that we can discover. Then we shall be able to plot a point every day and get, perhaps, 25 points in a month. Now, defects in cloth can be of various types: rough spots, holes, rips, and so on. I am not an expert on textiles, but I can spot defects.

Table 9

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
c	3	3	6	3	0	1	3	5	7	3	7	10	5	5	5	4	3	4	5	1	1	0	1	1	4

Here are the figures for 25 days. The control limits for the chart for c are simply this: the average number of defects c per sample plus-minus three times the square root of c (*cf.* Zl. 3 *Section* 16 page 19, and Fig. 4 page 20).

$$\text{Control limit...} c \pm 3\sqrt{c}$$

So we shall take the total number of defects $c=92$ for the twenty five days; divide by 25 to get the average, $c=3.68$; and obtain the square root of 3.68, $\sqrt{3.68}=1.92$. Then the control limits will be $3\,68 \pm 5.76$; therefore the upper limit will be 9.44

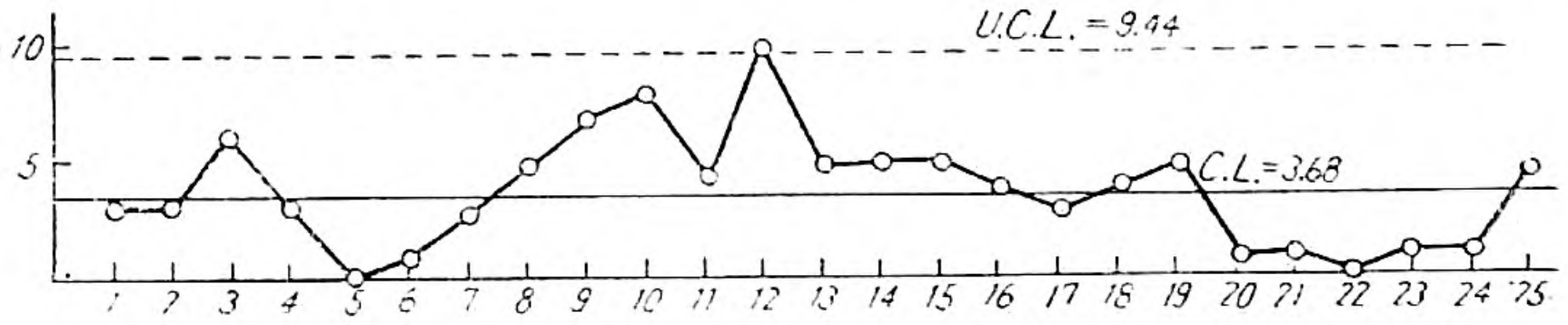


Fig. 18

and the lower limit is 0. So we shall scale off the chart up to 12.

This chart (Fig 18), you will probably agree, shows definite causes of uncontrolled variability. In the central portion I see probably 10 points that are above the central line. This is a pretty strong indication that something is wrong. Then from the 13th day there is a string of 7 points on about the same level, after which the points fall below the central line. I can only suggest that such trends call for investigation of unusual conditions. Check on what happened between 13 and 19, because there is good uniformity there.

Now I can give you as an example a control chart plotted for welded steel barrels supplied to General Electric Co. The number of defects were counted in lots of ten barrels. The average c was about 30. From this the control limits can be computed. Now, when the chart was plotted it was discovered that the process showed excellent control (Fig. 19). It had been supposed that there was a lot of variability in the barrels. But the control chart indicated that variability was apparently controlled. It was therefore decided that the manufacturer was doing as good a job as he could. So in order to reduce the average number of defects for a certain lot of barrels some fundamental change had to be made in the process.

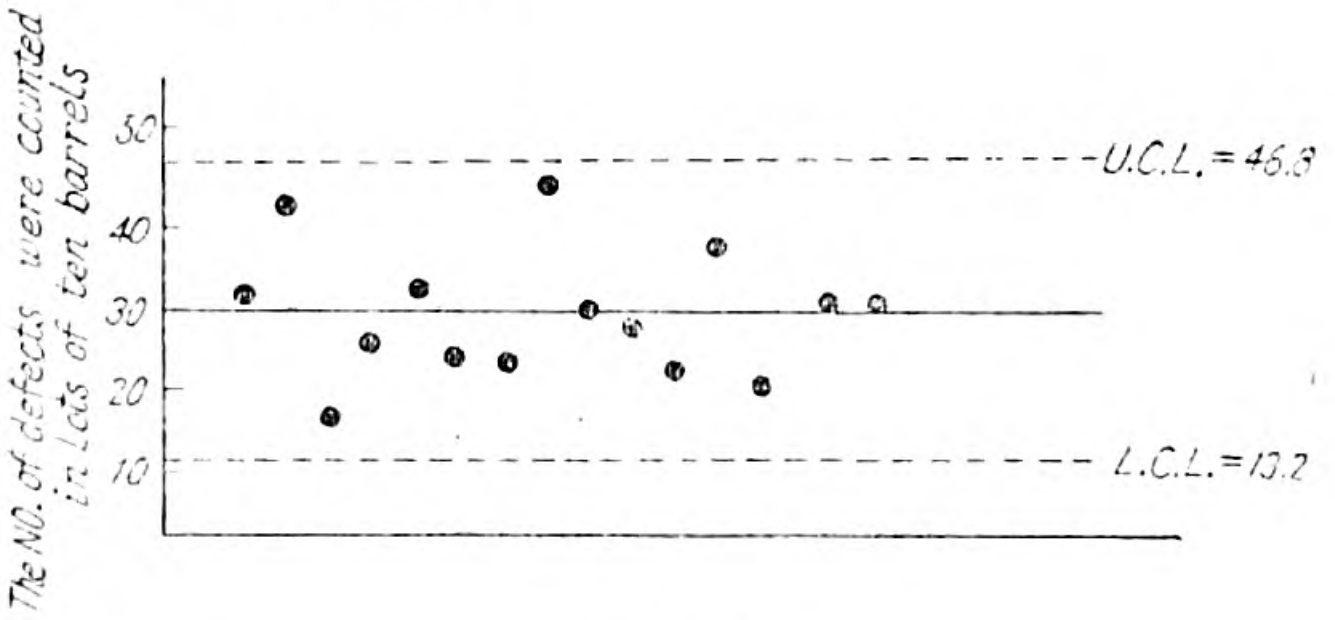


Fig. 19

The control chart for c is often used with finishes or paints. A sample area is scanned for the number of defects and a control chart is drawn up.

The distinguishing feature of the chart for c is that this symbol c stands, theoretically, for any number of defects. But it may also be used to denote the number of defectives in a large lot of articles, averaging 2,000 or 3,000 pieces. In your texts you will find on page 20 a chart from which you can read off the control limits for chart c. If you will look at Fig. 4 you will find that although the upper limit curves for 3-sigma and probability are almost identical, the lower limit curves differ

especially for smaller values for *c*. So when *c* is small I think it well to use the probability limits because these draw your attention oftener to identifiable causes of low quality.

Before we proceed, I wish to talk a little about further study. It is possible to do a great deal of work in these 8 days: we are making a lot of progress. But a great deal of further study, thinking, experience, and maturity is required. You must never stop studying; learn more. Every day you will find problems you will not be able to understand unless you continue studying week after week, month after month. Now, this can be done in various ways: you can read journals on quality control; you can get hold of good books on elementary statistics, or if you already have mastered that you can go on to more advanced studies.

Real study of *design of experiment* is not difficult. I do not mean to say that design of experiment is easy. But you can always begin learning something about design of experiment.

Some of you may find an opportunity to go to school and learn about statistical theory, both elementary and advanced, from good teachers. I know that here in Japan you are luckier than in other countries because you have many excellent teachers.

Then you can get books on quality control. Some are good and some are not so good. An author who is engaged in active statistical research will write good books; but one who has neither engaged in real work in a factory nor in theoretical research, can not produce books of worth. I am only trying to say that in any country there must be good books as well as some that are not good.

Now, I wish to point out that you can not know too much about mathematical statistics. Your knowledge of mathematical statistics simply can not be bounded. In the big cities there are evening courses for people who must work during the day. I

don't know whether you have them here. I think I was told that there is some institute for mathematical statistical research where government workers are taking courses. It may be possible to organize some excellent course in the evening; perhaps twice a week. Japan, I am happy to say, has a lot of competent teachers.

One thing you can do — that is very necessary for you to do —is to have meetings with each other wherever it is possible. I am sure that those of you who are in Tokyo could easily organize such meetings. Those in other industrial centers could do the same thing. These meetings and the groups that meet need not be at all formal; they can be very simply organized. All you need is someone willing to make the arrangements: find a speaker, notify the members of the group and fix some place. The problem of finding a suitable place may be difficult, I don't know. It depends on how many people gather together.

In America we held a number or 8 day courses like this one on quality control, beginning in 1942. In every case I talked to the students about the desirability of meeting together once or twice a month to discuss mutual problems. Such small groups were formed to hold regular meetings in Los Angeles. San Francisco, Chicago, Buffalo, Philadelphia, Boston, Detroit, Pittsburg, Cincinnati, and in all the cities where the 8 day courses were held.

The meetings had very humble beginnings. The discussions, too, were on a very low level; for there were many statistical problems that were not too well understood. But it isn't necessary to have a great deal of knowledge or be an expert in order to hold successful meetings of this sort. Moreover, in most cities there can usually be found a professor, or maybe several, who can be made to take an interest in statistical quality control. These people could assist you considerably. In America, actually 28

groups were first formed; and attendance at the regular meeting ranged from 10, 15, to about 200. Large numbers are not necessary.

The discussion might take the form of questions and answers regarding problems that a man has. Some competent professor might be asked to give some instruction on statistical theory.

Now, the informal groups that I told you were formed in the United States eventually organized themselves into the American Society for Quality Control. The strength of this association is seen in the journal that they issue; but even greater strength lies in the local discussion groups in the key cities.

Quality control had been known and used by a very limited number of manufactures in America for quite a while. Statistical quality control began in 1926; but until 1942 it wasn't used very much. Almost everybody had heard about it, but hardly anybody really knew what it was about. It can therefore be said that quality control in America began in 1942 with the 8 day courses which were given in that year and continued on to 1944, 45, and 46. Soon it was evident that something was happening: quality control was everywhere. Today quality control is used throughout most all the manufacturing factories of America. I don't mean just some one section of a plant; I mean throughout. Last night I was talking with a Japanese friend who has just returned from America. He did not go over to study manufacturing processes, but he had visited several plants. He heard a lot about quality control. He mentioned National Cash Register, Ford and quite a few other companies which I cannot recall; and he tells me there was talk and evidence of quality control everywhere.

I really believe that the most important thing you can learn at this 8 day course is to see that you *must continue to study*, and that mere study is not enough if quality control is to become

really effective. The rapid spread of quality control in America was through the local organizations which met frequently to talk and to study the problems that came up in actual practice. Sometimes they met in some room in a university, sometimes at a factory, sometimes at other places. It often happened that one manufacturing plant would invite the group to meet there. There are so many ways in which the meetings could be held that it is impossible for me to describe precisely how things were done. This afternoon I suggest you meet together and discuss what could be done, and decide. The main thing is to do something quick. After two or three years you will look back on this year 1950, and you will be able to see the result.

At first many discouraging things may happen and you may be disappointed. It was so in America when quality control began to spread. In many plants a man who wished to apply his learning in quality control could do so only outside of office hours. Every conceivable kind of discouragement and resistance was encountered. But with the tremendous advantages of quality control duly recognized after three or four years of uphill going there is now nothing but encouragement. You will have to be patient, and then your superintendent will come to learn the value of quality control.

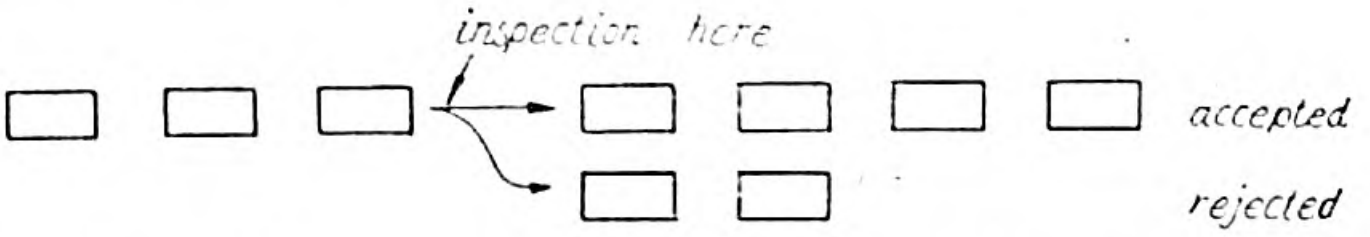
I can give you an actual example of what happened in an electrical appliance factory some 40 miles out of Los Angeles. The engineer who was anxious to work on quality control was not allowed to apply it during office hours. But he was a patient and persistent man. He kept working and working in the evenings after the plant had closed down for the day. After some time he discovered a cause of uncontrolled variability and reported what he had found. This led to the removal of the cause of variability and to considerable savings in cost for the company. The superintendent of the plant finally realized that this man had

a very useful tool. So from then on he was given encouragement and permitted to work on quality control problems throughout the plant. He worked on many different operations and in every case he as able to bring about improvements.

LECTURE 6

ACCEPTANCE SAMPLING

The topic which we shall go into this morning will be an introduction to "acceptance sampling." Now, in order to get a good idea of the subject I want you to have in mind a picture of what takes place in mass production of hundreds, perhaps hundreds of thousands, of articles which are as alike as it is possible to make them. Now, in the first place, this acceptance sampling is used only when mass production of thousands, or tens and hundreds of thousands of articles is undertaken. Think of a belt line, or conveyor, or a flow, a stream of articles which are going to be inspected at particular points. Now here comes a lot of articles. The size of this lot will be known as N. At the point where inspection is done a sample, whose size is denoted by small n, is drawn from each lot.



N.....Size of lot. n.....Size of sample drawn from N.

Fig. 20

At the point of inspection we apply a *rule of acceptance* which will separate the lots into two streams — accepted and rejected. Now, I am going to ask you a question: What kind of rule of acceptance would you like to have? From each lot N we draw a sample n, and on the basis of this sample we either accept or reject the lot. What kind of rule would you like to have?

Student: I would use a control chart and would reject any lot whose sample goes beyond the control limits.

Deming: Possibly such an idea might have merit.

One of the purposes of acceptance sampling should be to force the manufacturer to apply the control chart method to his process. Acceptance sampling in itself is not quality control during production. Acceptance sampling is sampling used by the purchaser to protect himself. But when the purchaser through his acceptance sampling rejects many lots he may be able to point out to the producer the improvement that should result from using the control chart method in the production process. However, the theory of acceptance sampling applies to any lot that comes to the point of inspection regardless of whether control charts were used in the production process. Therefore, what we would like to have now is a rule that will separate the good lots from the bad.

Let us assume we have a number of lots which by previous inspection we know to contain 4% defectives. Let us further assume that the lot size N is 500. Then in each lot we shall have 480 good articles and 20 defectives. Now suppose these lots are flowing past the inspection point at which we take a sample n; and suppose we apply the rule that if we find a defective in the sample we reject the lot, and if not we accept it. What will be the fraction defective p in the accepted lots? Has anybody an answer?

Student: There will be no change in the fraction defective of 4 per cent.

Deming: I agree. The answer is correct.

Now what has been accomplished by this inspection?

Student: ? ? ?

Then I will put the question in this way: Has this rule of acceptance done anything to improve the quality of the accepted lots?

Student: No. There is no change.

Deming: No change. That is right. I think we have a

very important lesson here. The rule of acceptance can only improve the quality of the accepted lots when incoming quality varies. For when incoming quality varies then there is the possibility that we might catch during inspection some of the worst lots and bring about an improvement in the quality of the accepted lots by rejecting the bad ones.

But to go back to the plan that we tried: that is, if we find a defective in a sample we reject the lot that it came from, but if not, we accept the lot. We have already learned that nothing is accomplished. If we send back the rejected lots to the manufacturer he may as well just send them back again to us on the chance that the samples we draw will not contain defectives. So the plan we now have does nothing except provide jobs for people working on the inspection. Then if something is to be accomplished we must have variability in the incoming quality, in which case you have a better chance of picking up a bad lot and rejecting it. Now we have come up against a very important lesson, and I want you to tell me what is wrong about what I am going to write on the blackboard. I want you to explain the mistake in this plan which was once widely used.

Lots are received in varying sizes (100, 500, 1,000, 5,000, etc.)
The inspector always draws and inspects a sample of 5% *from each lot.*
He accepts the lot if the sample contains less than 10% *defectives, and rejects the lot if otherwise.*

Now I will give you one hour to think over this exercise, and I will go on and give you another one. This one will take me about 45 minutes to explain, put certain tables and figures on the blackboard, and do the calculations. We shall use the same illustration that I gave to show the flow of lots, inspection point and the flow of accepted and rejected lots (Fig. 20). The inflowing lots will have varying quality—the fraction defectives p will

vary from one lot to the other. The good lots and the bad ones will be mixed up. At the point of inspection we apply a rule of acceptance: *accept the lot if no defective is found in the sample; reject it, otherwise.* This is not a good rule, there is something wrong. I want you to discover what is wrong.

I told you that the quality of the lots would vary. I shall tell you *how* that quality varies, but not *why*. This I shall give to you as a table which shows the distribution of the defectives (Table 10). Let us take trays for instance. You are the purchaser, and these trays come in lots of 50. Then we have three columns of figures:—

Table 10. Description of the incoming Lots

lot size $N=50$

Number of defective articles per lot	% defective p	No. of trays having quality p
0	0	100
1	2	200
2	4	400
3	6	200
4	8	100
5 or more	—	0

This table gives you a description of each incoming lot. But this makes little difference in the picture. At this point we inspect a sample whose size is n. Now the sample size n can be just 2, 3, or 4, 10, 20, etc. but never more than 50. Why not more than 50? Anybody?

Student: 50 as a sample size would mean 100% inspection, and you can not do more.

Deming: Now let us decide that n shall be 3 articles out of

each lot. Inspect 3 articles, and if we find no defects we accept the lot: if we find one or more defects we reject. That will be the rule of acceptance.

p is the fraction defective, whose distribution over 1,000 trays is found to be as follows:

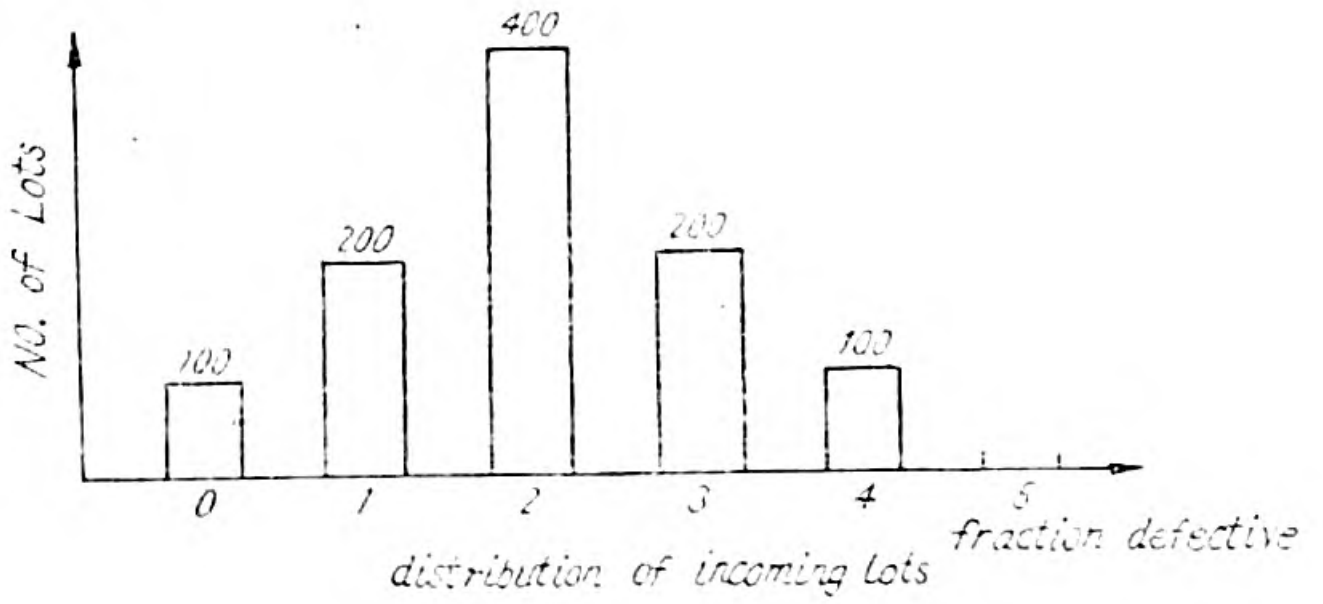


Fig. 21

Next, we take a sample of 3 articles out of each lot. We wish to calculate the number of lots rejected. Now let us suppose that the inspection is perfect, then with 0 percent defective the number of rejected trays will be 0. With calculation you will find that the number of lots rejected with $n=3$ will be: 0, 12, 46, 34, 22, 0; total 114. Now let us calculate the number of defectives in the lots rejected and the lots accepted (Table 11).

From the figures at the bottom of the column we calculate and find that the average percentage of defectives is 5.16 in the rejected lots and 3.85 in the accepted lots.

Well, it can be seen that the inspection and the rule of acceptance did not accomplish very much, because the incoming quality was 4% and the quallty in the accepted lots was raised only 0.15%. But partial separation was brought about because there were ups and downs in incoming quality—the accepted lots stream is a little better than the incoming stream. Now I want you to finish this exercise by calculating what the average of

defectives would be when the sample sizes are bigger, $n=5$, 10, etc.. See my book *Some Theory of Sampling*, page 287.

Table 11. ($n=3$)

p % defective	Number of trays	Number of lots rejected	*Number of defectives in the lots* rejected	accepted
0	100	0	0	0
2	200	12	12	188
4	400	46	92	708
6	200	34	102	498
8	100	22	88	312
10 or more	0	0	0	0
	1000	114	294	1706
Average % defective			5.16	3.85

Incoming average quality..... 4%

Average quality in the *Accepted Lot*..... 3.85%

When $n=3$

Now, will anyone make a guess about what will happen?

Student: The number of rejected lots will increase and the percentage of defectives in the accepted lots will grow smaller.

Deming: By how much? It might surprise you; but the figures tell us that there is not much improvement. This shows you the danger of surmising something without actual calculation.

n (sample size)	defectives in rejected lots	defectives in accepted lots
3	5.16%	3.85%
5	5.13	3.75
10	5.04	3.45

You will see that even 20% inspection improves the quality only from 4% to 3.45%.

Now, for many years in America the inspection and acceptance plan we have just studied was very commonly used. But you can see for yourselves that it was a complete failure. The only thing that such a plan is good for is to provide employment for a lot of people. It does not provide protection; it does not improve quality. It merely provides employment and raises the costs.

The lesson is clear: you had better know just what can be accomplished before adopting any particular acceptance plan. Today we have learned a great deal about acceptance sampling: you must learn what not to do as well as what to do.

We shall now go into some general remarks about acceptance sampling. It is an extremely interesting subject for study; it has fascinated some of the best mathematical writers in the world. A number of books has also appeared, among which can be found "Sequential Sampling" by Professor A. Wald of Columbia University. I cannot hope to cover a great deal in the short time we have here; and there is much of the theory of acceptance sampling that I have not yet studied. I have seen several very excellent books on the subject written by Japanese. I have seen two books still in the manuscript stage, which appeared to be excellent. Therefore I can leave you to study the theory of acceptance sampling after this course is finished.

I hope that you will not be discouraged; for some of the books look extremely difficult mathematically. I can say that the fundamental or basic principles of acceptance sampling are not at all difficult, but they do require a great deal of thought. This morning I explained some very important principles of acceptance sampling. You must always keep those basic principles in mind. Don't get lost in mathematics. Remember

in the first place that acceptance sampling is used by purchasers to protect themselves. The best protection is afforded by acceptance sampling done in conjunction with quality control at the manufacturing plant. It is not economical to try to get a good product by inspecting a lot and taking up only the best ones.

I want to return to the exercise that we have here. I think that you will agree with me that 20% inspection is very costly and furthermore ineffective. Therefore this is not a good plan. What should we do about it? What is the fundamental difficulty? Something is wrong. What would you suggest be done? Put it this way first: Obviously we are not getting anything from the plan. If we take a bigger sample, it will mean bigger cost of inspection. We already are burdened down with 20%, and to go in for fuller inspection will entail terrific cost.

The remedy is: BIGGER SAMPLES FROM MUCH BIGGER LOTS

But we have just pointed out that bigger samples will increase the cost of inspection. Yet bigger samples are the only answer. Under the circumstances what would you suggest? I will tell you. We want a bigger sample so the only way out is by taking bigger lots. Suppose you take N (lot size) = 1,000, then you will have, even with a sample of 50 (n=50), to pay only for the cost of inspecting 50/1,000—that is, 5% inspection.

With N=1,000 and n=50 we get good separation of good from the bad at fairly low expense. Even if we went ahead with 10% inspection with N=1,000 and n=100, the cost would not be prohibitive, while the separation would be excellent. But the use of bigger samples is only economical if you have bigger lots. This is a very important principle, but its application is not always easy. Your suppliers may not be able to give you a big enough quantity. Perhaps you are purchasing the same product

in small lots from several manufacturers. But in this case you can still protect yourself economically by combining the lots from the various manufacturers or combining several of the lots from the same supplier to make a big lot of, say, 1,000. Nevertheless this not always easy to do, nor is it always the correct thing to do. You will be mixing up bad quality from a poor manufacturer with good quality from a good manufacturer. One of the purposes of acceptance sampling is to reject a portion of bad lots from a bad manufacturer and turn them back. But if you mix up the good and the bad you have no indication that any one manufacturer is so bad as to have to get him to take back the bad material. Now I would have several answers to the problem. A number of them are satisfactory. From the theoretical standpoint of acceptance sampling it may be desirable to mix up the unequal qualities to make up a suitable lot size; but this would be a bad thing to do from the standpoint of encouragement.

The two purposes of acceptance sampling are:

(a) To protect yourself against accepting poor quality. This purpose is fully treated, mathematically and otherwise, in the books that have been written on acceptance sampling. But there is another purpose which is very difficult when it comes to explanation. It varies greatly with some articles, especially finished products; and there is no way of handling it completely mathematically. But you must always keep in mind the mathematical principles.

(b) To encourage the manufacturer of good quality and penalize the manufacturer of bad quality. One way of accomplishing this purpose (b) is to sort, screen or rework the rejected lots at the expense of the producer. But I don't like that very well. The manufacturer of bad quality will have the additional expenses of sorting and reworking. Now I have a question: Is

the producer's expense any problem to you as a purchaser? Do you care anything about the manufacturer's cost?—this not worked out by mathematics.

Student: It varies with the circumstances. In certain phases of economic development, such as the situation we have now in here in Japan where the average consumer has very little purchasing power, it may be more profitable to do away with inspection entirely and let the consumer benefit from the savings in cost. But from the ideal standpoint I believe there should be some point of agreement between the producer and the purchaser.

Deming: The answer is very good. The total cost of the product must include the cost of inspection, even though the maker of the article pays for it. Yet I believe if you are looking for perfect economy there should be some way of encouraging good quality and penalizing bad. I am a very poor economist. As for agreement between the producer and purchaser (consumer or user), I am sure this is very necessary. If the raw materials that I have to work on come in from some other department of the same plant, or are supplied by some other manufacturer, and these are all scraps and defectives, you must try to do something about it. One of the main tasks that you must perform is to learn something about the people who supply you. If an outside supplier is giving you trouble, bring him into your plant and show him quality control; give him a book on quality control; tell him about your difficulties; show him how he can help you, and how he needs quality control.

Next I want to tell you something about producers' and consumers' risks, that is, in acceptance sampling. The consumer's A risk is the probability that a lot having an unwanted quality P_1 will be accepted. You have an acceptance plan. A shipment comes in for inspection, then you divide the lots into accepted and rejected. Now, sometimes the unwanted P_1 will get into the

accepted stream. P_1 is unwanted quality, but once in a while you are going to accept it anyhow. By any plan for acceptance without 100% inspection you will at times accept P_1. How are you going to reduce this risk to zero? With 100% inspection — inspect every article and reduce this unwanted article to zero.

The consumer's B risk is the probability that a lot, drawn at random from a supply of quality P_1 will be accepted. How can you reduce this consumer's risk B to zero? With 100% inspection.

Producers have risks too. The producer's A risk is the probability that a lot of good quality P_0 might be rejected. P_0 is desired quality. There is also the producer's B risk. It is the probability that a lot, drawn at random from a supply of quality P_0, will be rejected.

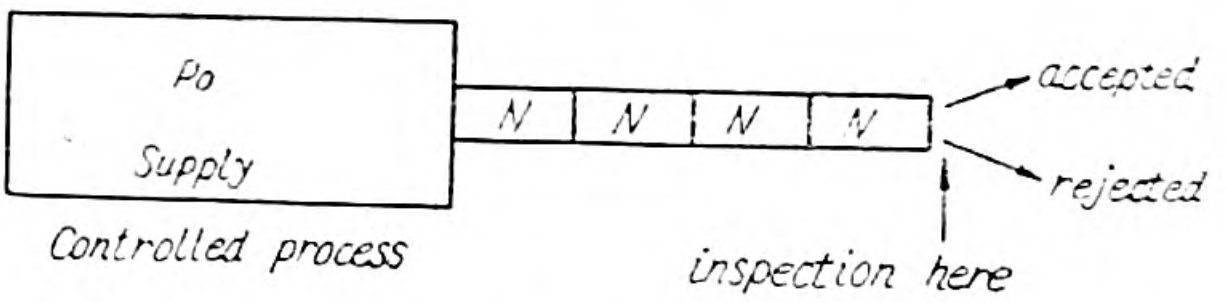


Fig. 22

The point of inspection is where you draw samples, and where, by inspection of the samples and application of a rule of acceptance, you accept or reject the lot. In applying the rule of acceptance you will have four risks: A producer has a product of quality P_0, perhaps with only one half percent defective—very good quality. Even though he has a good quality, it will happen that the lots, sampled at random, will sometimes be accepted or rejected, because the lots and the samples vary in quality. Sometimes, one of the lots is going to be rejected.

Now, I am going to deal only with the consumer's A risk and the producer's B risk. For I think these are the most important. Tell me this: How can you reduce the producer's B risk to

zero? The answer is that you can not do it. Will 100% inspection do? No. In 100% inspection you make the sample size equal to the lot size. But that will not reduce the producer's B risk to zero. Some of these lots have quality P_1 which is not desired. I remind you again; these lots of size N are drawn at random from the supply and have varying quality. They may have quality P_1 or worse. You inspect 100%, then the lot will be rejected, but it is only going to happen once in a while. We can make this risk vary from 10% to 1% by increasing the size of the lot. But we can not make it zero.

The consumer's A risk is the risk of accepting a lot which has the undesired quality P_1.

That is as far as I can go at this time. You will need quite a deal more of time to proceed further. You continue studying. I think however that this is a good place to stop, because I wish you to think about these four risks. These risks were known and have been dealt with since 1924 by Dodge and Shewhart. According to the Dodge and Romig table the consumer's risk is the A risk, and the producer's is the B risk. A lot of people have misunderstood their writings and tables.

I mentioned yesterday that a plan of acceptance must be in accordance with a definite rule.

A PLAN OF ACCEPTANCE is determined by:

N, the lot size

and, in the case of single sampling, it is determined also by:

n, the sample size, and
c, the acceptance number.

In the case of double sampling.

n_1, n_2, the sample sizes, and
c, c_2, the acceptance numbers.

Further, the PLAN involves:

α, the producer's B risk

β, the consumer's A risk

Now, we are dealing with a single sampling plan. There is a number c, likewise a number n which refers to the size of the samples. There will also be double sampling plans in which we have n_1, n_2, and acceptance numbers c_1, c_2.

Any sampling plan must follow a certain rule. And this rule involves the producer's B risk α and the consmer's A risk β, which I explained to you yesterday.

Further, any acceptance plan will have what is known as an OC curve. OC means *Operating Characteristics.*

Now I will plot two curves showing the probability of acceptance.

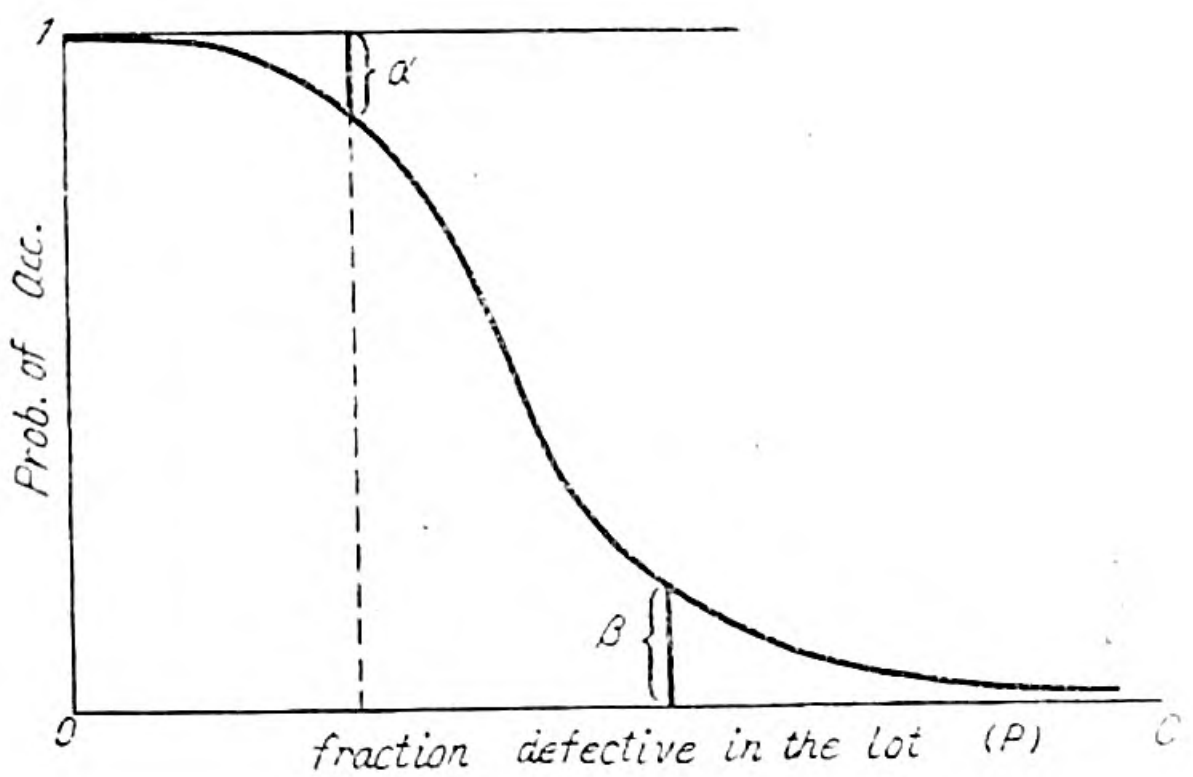


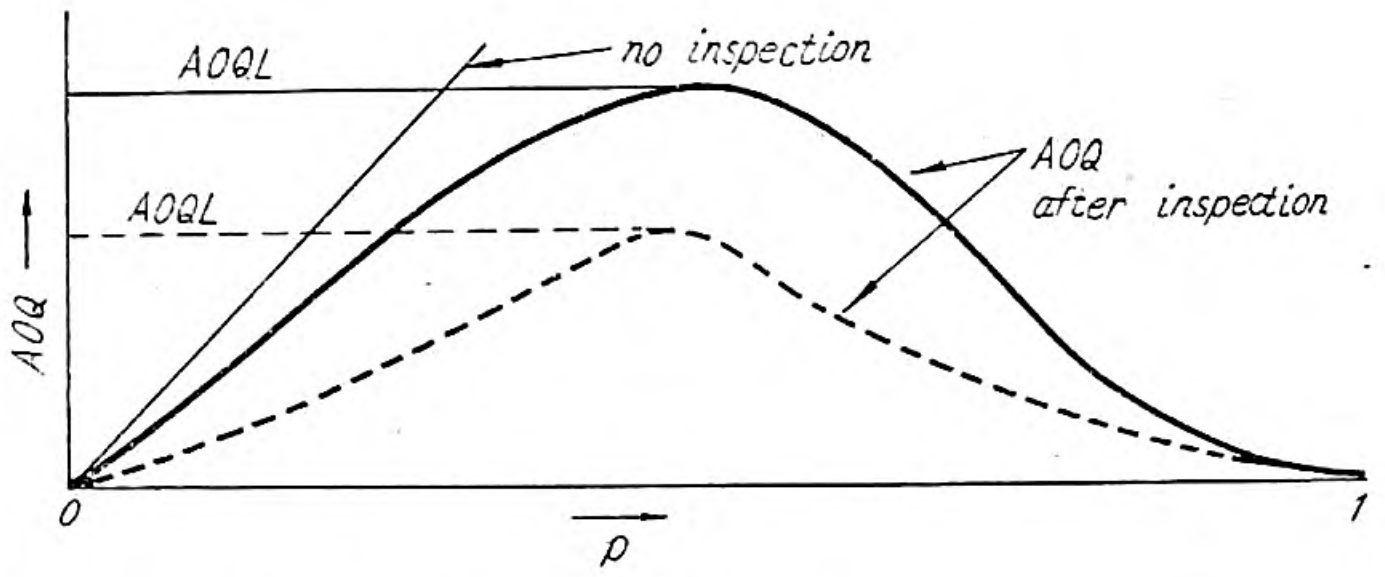
Fig. 23

The curve for the consumer's A risk β begins from zero, the point of perfect quality, and goes up to where there will be a certainty of acceptance. Now, for any particular quality there will be a distance indicating the producer's B risk, α. I told you yesterday that it is possible to reduce the consumer's A risk to zero by perfect inspection of each p. But α can not be reduced to

zero. The producer's B risk can not be reduced even by examining every article because even if the producer has controlled quality at the desired level, some of the lots will be rejected. That is the producer's B risk. I explained it before.

Now, I shall make a remark: No one is expert enough to judge immediately by looking at any particular OC whether the curve arises from single, double, or sequential sampling. Another way of saying this is that any curve on either of the above two diagrams can be closely approximated with either single, double, or sequential sampling plans.

Next, I ask you to think about another kind of curve. There is a curve that give the *average outgoing quality* and it is known as the AOQ curve.



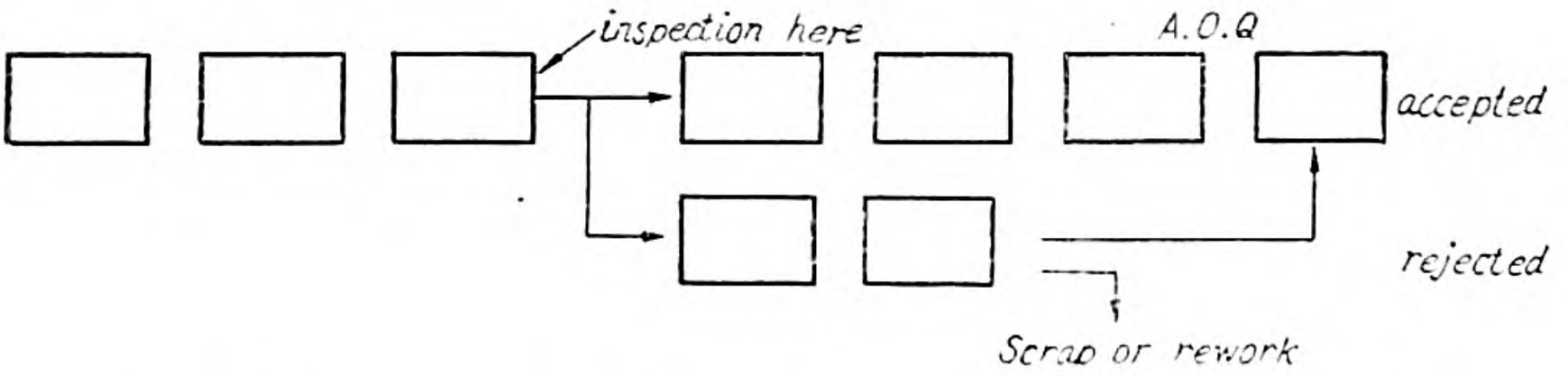
AOQ . . . Average Outgoing Quality
AOQL . . Average Outgoing Quality Limit (Max. AOQ)

Flg. 24

Look at the diagrams. The lots of fraction defective p will be coming in a stream; then we have the inspection point where some lots will be accepted and some rejected. The average outgoing quality can then be plotted as a curve, with AOQ zero when the fraction defective is zero and unity.

I must introduce a slight complication. The rejected lots are to be resorted to separate the good articles from the bad, or to rework bad ones; then these articles from the rejected

lots are to be sent into the accepted stream.



Now, you will see that if there is perfect quality in the lots, the AOQ will be perfect quality. In other words, we start from zero. When the quality is not perfect, p is not zero, so the average AOQ too will not be zero. If p is 1 or near to 1, then the quality is very bad; and under the circumstances most of the lots will be rejected. Now the maximum AOQ will be AOQL, the average outgoing quality limit. Now any plan of acceptance, with its n_1 n_2 c_1 c_2, and the rule about sorting or screening, and returning the good articles to the accepted stream, will show up as a certain curve on this diagram. Some other plan will have a different curve, perhaps with a different AOQL. Therefore we may designate any acceptance plan either by α and β, which come from n and c, or by its AOQL and its producer's B risk.

With the famous "Dodge-Romig Sampling Inspection Table" which came out in final form in 1944, but which had been in use previously for many years, you will find the tables headed "Consumer's A Risk" will also show the AOQL. Now when in some of the tables they designate AOQL as the heading. and also the consumer's A risk, which they designate by the letter p_t, the values of α are not shown. That is to say, the producer's B risk is not indicated. However this can be calculated.

The Dodge-Romig tables have been worked out to give the consumer's A risk (β and p_t) and AOQL at minimum total cost of

inspection. Now there are other tables by other people for other purposes. Some of these do not aim at minimum cost. This you must remember when using such tables.

Now to go on a little with theory, I should like to show you a diagram which has been very helpful to me in thinking about acceptance sampling. This is called the Diagram of the Random Walk (Fig. 25).

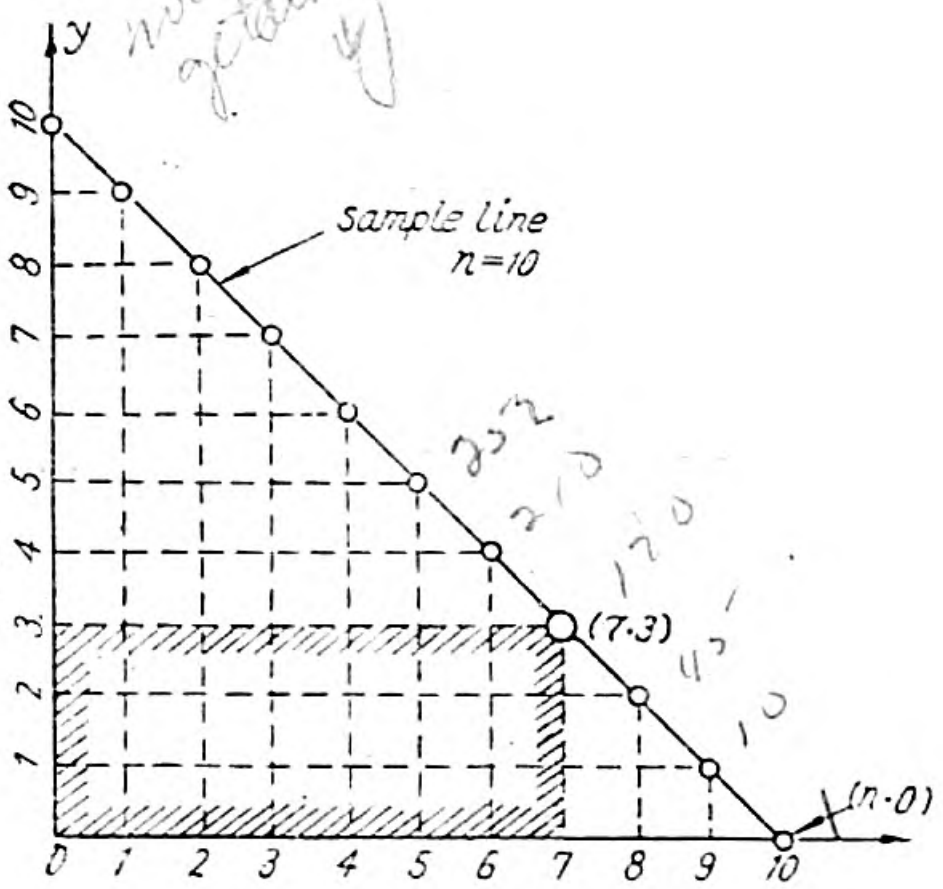


Fig. 25. Diagram of the Random Walk

Single sampling plan

n. c. N.

x. . the number of good articles in the sample

y. . the number of bad articles in sample

$x+y=n$

suppose $x=7$ and $y=3$

First of all I will talk about a single sampling plan. Then we shall have n, c, and N. Now, what would be $x+y$?

Student: It would be small n.

Deming: Very good, it is n. Now, how many of you have studied geometry? What would $x+y=n$, stand for?

Student: A straight line.

Deming: Very good.

Now we take ten points along the horizontal line and the same number along the vertical. Now just suppose that x is 7 and y is 3 for a particular sample; then the point (7, 3) can be taken to represent that sample, and other points will represent other samples of different values for x and y.

Now suppose I draw a sample—one article—, and I test it; then every time I draw such a sample I move a pawn to the right or upward like this:—

1. Draw an article 2. Inspect the article 3. Move your pawn—

1 unit → if good, and
1 unit ↑ if bad.

Suppose I start with the pawn down here at the left-hand corner (0, 0), and draw an article every hour. I inspect the article, and if it is good I move the pawn one unit to the right; if bad, one unit up. I will repeat that 10 times, or generally n times. Then we have a sample of size n.

Now, where will the pawn be when we have drawn all the articles, tested them and recorded them? Of course the pawn must be somewhere, but where?

Student: It will be on that straight line.

That is correct. So we call that the *sample line*; the pawn must fall on that line. Tell me, suppose that in a sample I have found 7 good articles and 3 bad ones, how would the pawn get there?

Student: It would be a zigzag course.

Yes. Now, if we draw a rectangle, using points (0,0), (0,7), (7,3), and (0,3), all the possible ways for the pawn to get to (7, 3) will be confined in that rectangle. There are a number of paths from (0, 0) to (x, y) and the following formula gives the answer:

$$\binom{n}{x}=\binom{n}{y}=\frac{n!}{x!\,y!}$$

Now, each path has the same probability; but before we go into that I want to ask you a question: How many ways are there from the point of origin out to point $(n, 0)$?

There is only one: $x=n$

and $y=0$ then, $\dfrac{n!}{n!\,0!}=1$

if $x=n-1$

and $y=1$ then, $\dfrac{n!}{(n-1)!1!}=n$

What then is the probability for any one path from 0 to (x, y)? Remember there are two ways of drawing articles for the sampling: Take an article from the lot, move the pawn and put the article back in the lot; or, do not put it back into the lot. If the article is drawn with replacement, the probability is $q^x p^y$. Now this will be the probability that we must use for the producer's B risk.

Now, I am going to ask another question — a new one although it is almost the same. What is the probability for all paths from 0 to (x, y)? We will have a different answer which is a summation of the answers for the first question. The answer, if drawn without replacement, is this:

$$\frac{\binom{Np}{y}\binom{Nq}{x}}{\binom{N}{n}}$$

I want to make some very important observations here.

Note 1. For calculating the producer's B risk, you use the probability $\binom{n}{x}q^x p^y$. The lot size N is not present. It goes to show that the best the producer can do is to control his process, that some of his lots will still be rejected even when the quality level is what he wants.

Note 2. For calculating the consumer's A risk, you use the term just printed above.

Now, I realize that some of you may have not studied any theory, but I do not want you to worry about that. As a matter of fact you can now say that you have studied some-

thing about the theory of probability.

To get back to the diagram, I shall put in two lines. The lower line indicates quality that is good enough, while the upper line shows quality that we cannot accept.

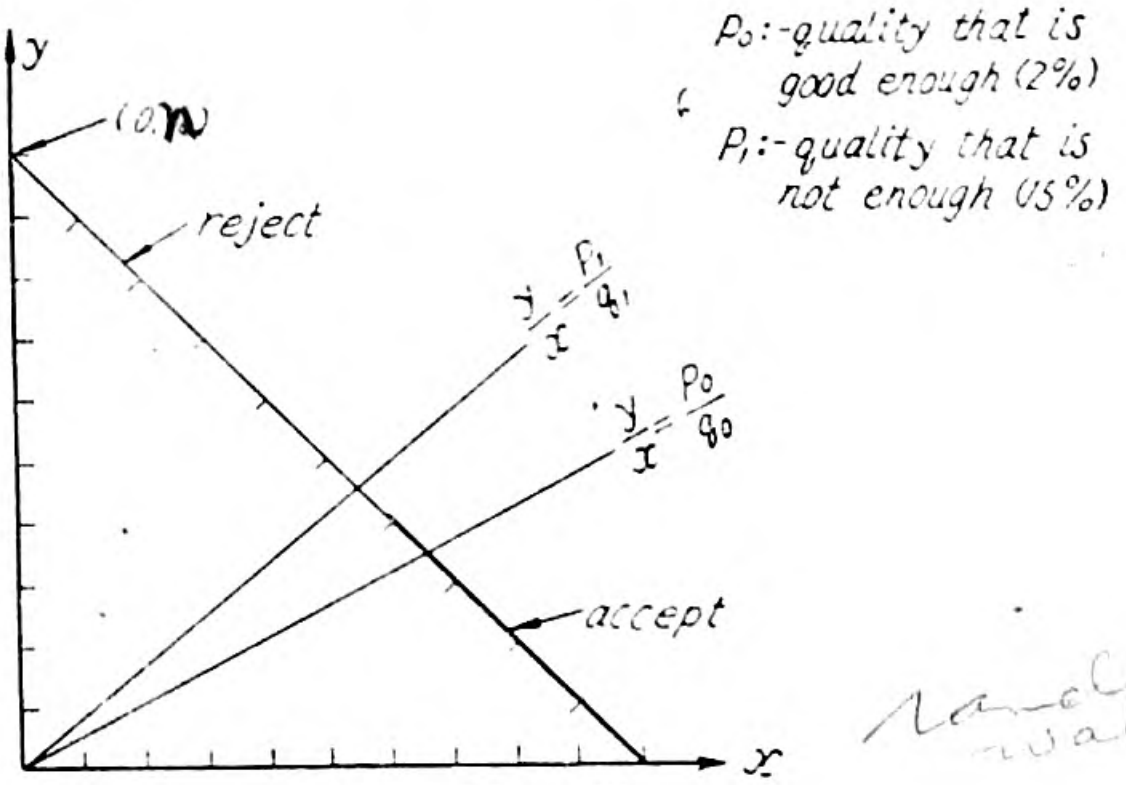


Fig. 26

Now remember that we are dealing with calculation of the risks in a single sampling plan. At what points shall we reject the lot? The RULE OF ACCEPTANCE says: "Reject the lot if the pawn falls at a certain point (or points)." So we come to the following inequality:—

$$\beta \geqq \sum_{y=0}^{c} \frac{\binom{Np_1}{y}\binom{Nq_1}{x}}{\binom{N}{n}}$$

Let c be the *acceptance number* as before. To find c, calculate it from the inequality. Start from the bottom and keep on going. Stop when you get just the right equality. At the bottom, y is zero and x is n. It is just plain arithmetic.

Now if you continue in this way, you might get β equals 0.1. Again start at the bottom; take the next term; calculate the next point, add it, and so on. Before the point exceeds β, stop. Otherwise proceed and add up until y equals c. Do not exceed β. *If c is the* acceptance number, then the rule is this:

Accept if $y = 0, 1, \cdots\cdots, c$

Reject if $y \geqq c + 1$

We can now mark off the lower part of the sample line and call it the REGION OF ACCEPTANCE (Fig. 26). We accept the lot if the sample falls anywhere in the region of acceptance. We reject it if the sample falls in the region of rejection. To decrease the consumer's risk we pull the region of rejection farther down.

CALCULATION of α, the producer's B risk.

$$\alpha = \sum_{y=c+1}^{n} \binom{n}{x} q^x p^y$$

We take these probabilities in the same way as those that we worked out before. Calculate them beginning from the top, then come down all the way to the region of rejection. Or, we will start from $c+1$, the lower part of the region of rejection, and go on up.

Sometimes it happens that in order that the consumer's risk A exceed the producer's risk B, c must be zero. Then you reject the lot if $c=1$ or more.

What I have just told you is a very brief sketch of the theory of acceptance sampling.

Now, I want to say a few words about DOUBLE SAMPLING. In double sampling we take two samples. Look at this diagram (Fig. 27).

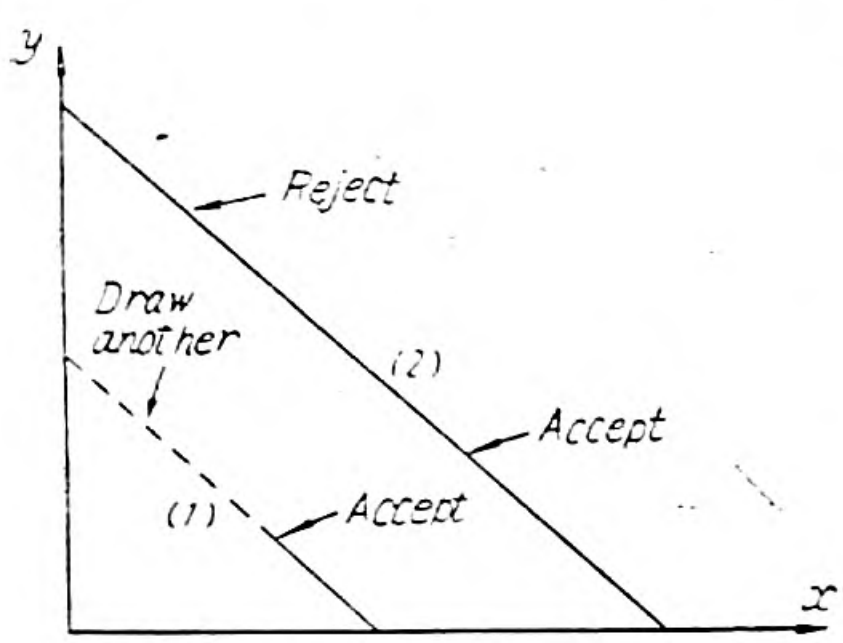


Fig. 27

We take two samples: one first, then perhaps another one. Now the first line represents the first sample n. This we divide into two parts: "accept" and "draw another." The second sample n_2 is divided into accep-

tance and rejection. Then we have two acceptance numbers c_1 and c_2. The rule is: accept the lot if the number of defects y_1 is equal to or less than c_1; otherwise make a second sampling. On the second sampling, accept the lot if y_1 plus y_2 is equal to or less than c_2; otherwise reject. In other words, in double sampling we never reject a lot on a single sampling. One sampling is enough for acceptance, but we reject only those which have failed two samplings.

Double sampling has these advantages: (1) slightly smaller total amount of inspecting than in single sampling; (2) the producer thinks that he is getting a better deal, as he is not rejected in a single sampling.

Actually it isn't so. The producer *is not* getting a better deal. Remember I told you that for any sampling plan that we make up, there will be a double sampling plan that has the same characteristic as the single sampling plan.

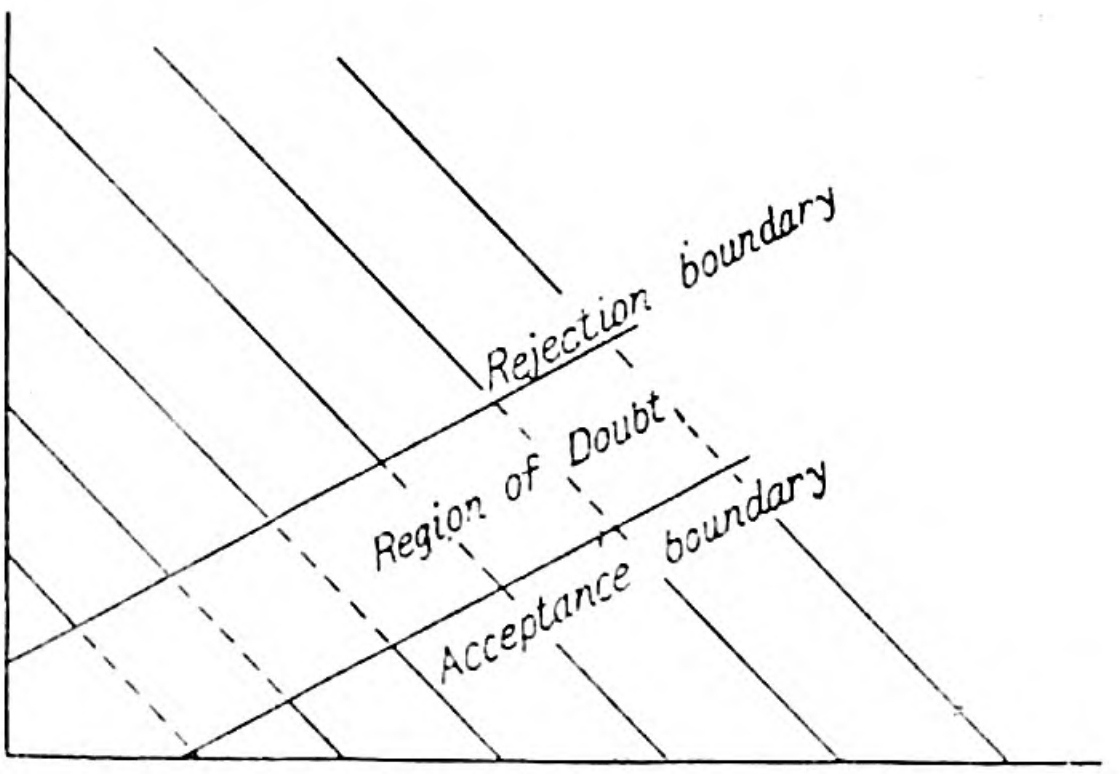


Fig. 28

We can produce the desired α and β in either the single or double sampling plans. Only, in double sampling we have a slightly smaller total amount of inspection. There is one disadvantage in the double sampling plan. It is a little more

complicated, and you must train your workers.

Now, I will go on the SEQUENTIAL SAMPLING. I shall start in this manner.

The idea of this sequential plan is just an extension of double sampling. Take a sample, say, of 5 or 10. Imagine the whole series of samples sizes which could be drawn on this diagram (Fig. 27 and 28). You reject the sample if the point ever falls above the reject line. The position and tilt of the lines will determine α and β. The equations of the lines are:

Rejection boundary ; $$\left(\frac{q_1}{q_0}\right)^x\left(\frac{p_1}{p_0}\right)^y = \frac{1-\beta}{\alpha}$$

Acceptance boundary ; $$\left(\frac{q_1}{q_0}\right)^x\left(\frac{p_1}{p_0}\right)^y = \frac{\beta}{1-\alpha}$$

These lines are parallel, and β determine their separation and tilt. The probability theory for the sequential plan is usually considered pretty difficult, but I have found an easy way to work it out.

With my way of doing it is easy and done in a single page. Today however we will not go into the calculations. These two boundaries are lines: $ax+by=k_1$ represents a line, and $ax+by=k_2$ is another line parallel to it. Take the logarithms on both sides. Then all you need do for calculation is simply to add up the probability of reject or acceptance. I don't have the time to explain it here on the blackboard, but actually it is very easy.

Now, in sequential sampling you draw a minimum sample of 5 or 10 articles. If the number of defectives falls in the region of doubt, you take some articles at random from the lot and test them until the pawn falls on the rejection boundary or the acceptance boundary—not in the region of doubt. Just start from the minimum sample. Somewhere along the line of samples zigzagging here and there will be a point where this or that boundary will be touched. If you keep on drawing and testing,

the lot will either be accepted or rejected.

In testing, the inspectors have tables containing figures in front of them. There are acceptance numbers and rejection numbers. When these are reached, the job is finished: the lot is either accepted or rejected.

Don't go away with mistakes in your way of thinking about sequential sampling. Some people in America have come to think that sequential sampling is the ultimate attainable improvement of double sampling. This is not true. Sequential sampling, under certain conditions, may be more practicable. When the incoming quality is just about what is wanted, or anywhere in the vicinity of p, sequential sampling is better than double sampling. Sequential sampling should thus be used when the cost of inspection or testing is very great. Then it may bring about substantial savings. Where the cost of inspection is very low, I don't think you will wish to use sequential sampling. You should do some calculation first: perhaps single sampling is the simplest and just about as economical as any.

Sequential sampling is used by many. But I think that most inspection is done by the single or double sampling method. There is a great deal more one can say about acceptance sampling. I have told you only some of the most important points.

As I said at the very beginning of the first day, you will have to continue your studying. I told you then that the knowledge of statistical techniques means great power. That power must be continuously replenished. You must never let your studies stop. You must have regular discussions and meetings as I suggested; otherwise that knowledge will wither and die like a plant. I believe that the power which started with this course you will continue to nourish with further study, meetings and reading; and that it will become a great asset for Japan—a power for better living, for happier living, for peace,

for the good of the world. Because of these techniques, with the help of these techniques, it is possible to build superior quality which in time will be respected all over the world. Remember, I told you yesterday you must be patient. You must teach your colleagues, your suppliers, your superintendents, your foremen, both in your own plants and others. You must get them started on quality. From your reading of journals on quality control and other statistical, engineering literature, you will see that the technique of statistical quality control has a wide variety of uses. Physicians use the method for studying disease. One doctor divided the San Francisco area into districts of 50,000 people. He found that in some districts there was a distinctly higher incidence of disease. He used the *c*-chart and by so doing found the reason why. Some districts where the frequency of disease was high were those where the water mains were polluted or the fish market was not hygienic. It turned out to be a very valuable study because the causes were found and removed; and when again a survey was made, the formerly bad districts had come within the control limits.

The *c*-chart is sometimes used for study of accident rates—the number of accidents, say, in your plant, month after month. If there are going to be any accidents, the total number will vary from week to week; but if in your study you find all the points within the control limits, you do not stay satisfied but make some fundamental changes — install safety screens, make use of modern color schemes, etcetera — and change the control limits.

The control chart can also be used to check on the cost of products from suppliers or various departments of a plant. Sometimes you will find the cost of a product way above the limit. Then there is good reason for investigation, and the result will be economy.

There are many many uses of the control chart that I have

not mentioned, which you may not even have dreamed about. At any rate, your success will depend upon your knowledge, patience and your ability to utilize the control chart in different ways. No two problems are alike. In other words you will have to be a really good engineer. The word "engineer" comes from the word "ingenuity." In French for instance, these two words are almost identical. You must be equipped to think and to put your thinking into action in a practical way.

Now, I wish to say that the performance of the students in this course is the best that I have ever experienced. I feel very much encouraged. I think that you have done remarkably good work.